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基于分形理论的新安江模型参数空间尺度效应分析

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摘要：水文尺度效应一直是当今水文科学研究的前沿问题。基于分形理论建立模型空间敏感参数在不同尺度之间的定量转换方程，进而提出了一种水文模型参数空间尺度效应分析方法。以三水源新安江模型为例，分析模型参数在嘉陵江小河坝站以上流域的空间尺度效应。结果表明：新安江模型空间敏感参数（SM、KG、KI、CG、CI、CS）具有随空间尺度变化的标度不变性，与流域集水面积之间定量关系可用幂函数关系进行描述，并可建立相应的定量转换方程。通过在研究流域不同空间尺度上的应用检验，发现基于分形理论标度不变性建立的新安江模型参数空间尺度转换方程具有较高的适用性，能实现有资料地区模型参数向无资料地区的移用，为无资料地区水文模型参数的确定提供参考。

关键词：水文空间尺度；新安江模型；分形理论；小河坝流域

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水文现象在不同的时空尺度上可能会表现出不同特征，而其背后又往往隐含着统一的规律性；加强水文尺度效应研究，建立不同尺度之间的转换关系，不仅可以加深对水文过程的认识，还可以解决水文信息由有资料地区向无资料地区以及不同时空尺度之间的移用问题[1]。因此，研究水文尺度问题具有重要的理论意义和实用价值，也一直是水文科学研究的前言[2-4]。

早期研究水文尺度问题主要采用的是统计相关（回归）类方法，如国内外普遍采用的洪峰经验公式等[5]。近年来，混沌、分形、小波分析等新理论新方法开始被引入水文尺度问题的研究与应用。Sivakumar[6]首次应用混沌理论研究降雨量的尺度效应，结果表明不同时间尺度降雨数据权重的变化具有混沌性，进而提出了一种解决降雨量降尺度问题的混沌模型。Kumar[7-8]综合分形理论与小波分析，提出了一种分离空间降雨场大尺度和小尺度特征的方法，表明降雨波动可以近似为稳定分布。Gupta等[9]在洪水区域分析中引入标度不变性假设，为研究不同尺度水文过程的空间变异性提供了基础。在国内，王文圣等[10]利用小波变换时频局部化功能，将水文序列的频率特征在时间域上进行展现，得到各种周期的强弱和分布情况以及突变点。李贤彬[11]利用分形与小波耦合方法，开展汛期日径流过程预报，结果优于级差分析法与分维估计法。常福宣等[12]利用分形理论对嘉陵江流域的洪峰和面积之间进行尺度分析，发现洪峰随流域面积的变化具有标度不变性。

前述研究大多是针对水文变量（如降雨量、洪峰流量等）的尺度问题进行研究，而水文模型参数尺度效应的研究并不多。本文选择嘉陵江小河坝站以上流域，基于分形理论，研究新安江模型参数的空间尺度标度不变性，以期揭示模型参数的空间尺度效应，建立模型参数在不同空间尺度之间的转换关系，解决无资料地区水文模型参数的移用问题。

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1　研究方法

1.1　分形理论及其标度不变性

Bloschl 和 Sivapalan[1]曾提出研究尺度问题主要有两种方法，一是基于建模的模型导向方法，二是基于维度分析和相似性概念的系统整体性方法，主要涉及到量纲分析和分形的概念，其中分形因其能以简单的方式处理复杂过程而被广泛用于水文尺度分析。

分形理论由 Mandelbrot 在19世纪70年代中期创立，是指组成整体的各个部分，其形态以某种方式与整体相似，具有自相似性和标度不变性[13]。在数学角度上，分形理论中的自相似性意味着标度不变性。具有标度不变性的研究对象，在一定的尺度范围内，它的某种属性在不同尺度之间的关系可由仅与尺度比值有关的标度变换所决定，该尺度范围称为标度区间，超出标度区间则变量可能不再满足标度不变性。因此，若某水文变量满足标度不变性，则可通过标度变换实现不同尺度下的水文特征(或规律)推求。

定义随机场$\{Y(x)|x\in R^d\}$，其中x为时间、一维或多维空间或时空，d为此空间的维数。对任一常数$\lambda>0$，定义$Y_\lambda(x)=Y(\lambda x)$，若$Y(\lambda x)$与$\lambda^\alpha Y(x)$具有相同的概率分布函数，即

$$\{Y(\lambda x)\}\underline{\underline{dis}}\lambda^{\alpha}\{Y(x)\} \tag{1}$$

则称$Y(x)$满足标度不变性，或具有标度性质，指数α称为标度指数[9]。式(1)表示对任一概率P，有

$$Y(\lambda x)=\lambda^\alpha Y(x) \tag{2}$$

或

$$\log(Y(\lambda x))=\alpha\log\lambda+\log Y(x) \tag{3}$$

标度不变性是分形的重要特征，本文利用分形理论的这一特性，在具有相似的地理和气候条件(或水文一致性区)的流域中选取不同面积大小的子流域，分析流域水文模型参数的空间尺度效应，确定水文模型参数是否随空间尺度变化而具有标度不变性。对具有标度不变性的模型参数，即可实现模型参数在不同空间尺度之间的转换。

1.2　模型参数标度不变性分析方法

在实际应用中，可以通过在若干不同空间尺度、具有水文相似性的流域上建立某水文模型参数的标度不变性分析成果，将在有资料流域空间上率定好的模型参数移用至无资料流域空间，主要流程步骤如下。

(1)选择嵌套式水文相似性对比流域。针对某一无资料目标流域，选择空间嵌套式、具有水文相似性的若干对比流域，见图1。

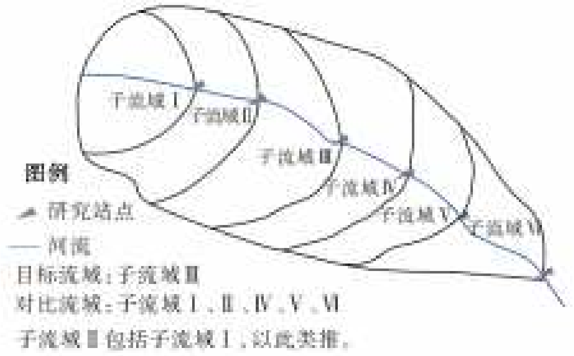


图1　空间流域示意图

(2)率定水文模型参数。在空间嵌套的若干流域上，利用气象水文数据构建水文模型，率定出各流域上的模型参数。

(3)建立模型参数与流域面积定量关系。以面积最小子流域模型参数值θ_0为真值，对应流域面积F_0；各子流域模型参数值为θ_i，对应流域面积F_i。以各子流域模型参数θ_i与真值θ_0的比值即θ_i/θ_0为纵坐标，相应流域面积F_i/F_0作横坐标，建立模型参数与流域面积的关系曲线$\theta_i/\theta_0\sim F_i/F_0$见图2。根据关系曲线选出水文模型的空间敏感参数，并利用数理统计方法建立两者间定量关系$\theta_i/\theta_0=f(F_i/F_0)$。

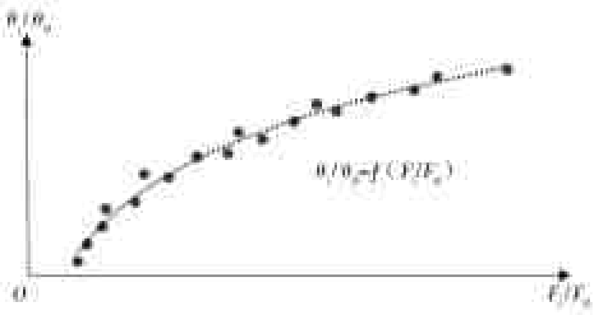


图2　模型参数与流域面积关系曲线

(4)模型参数空间标度不变性检验。设$\theta(F_j)$为流域面积F_j上随空间尺度变化的敏感模型参数，它受多种因素影响，具有较大不确定性，假设其为一随机变量，并设尺度F_i为F_j的λ倍

$$F_i=\lambda F_j \tag{4}$$

类似于$\theta(F_j)$，$\theta(F_i)$作为尺度F_i所对应的的随机变量。假设

$$\theta(F_i)=\lambda^\alpha\theta(F_j) \tag{5}$$

其中α为一常数，根据统计学中随机变量分布密度函数的相关知识，设$\theta(F_i)$分布密度函数为

$f_i(\theta_i)$，$\theta(F_j)$分布密度函数为 $f_j(\theta_j)$，则有

$$f_i(\theta_i)=f_j(g^{-1}(\theta(F_i)))\times\frac{dg^{-1}(\theta(F_j))}{d\theta(F_j)}=f_j\left(\frac{\theta(F_i)}{\lambda^{\alpha}}\right)\times\lambda^{\alpha} \tag{6}$$

即存在

$$\theta(F_i)\underline{\underline{dis}}\lambda^{\alpha}\theta(F_j) \tag{7}$$

根据标度不变性的基本公式(1)，可得此时流域模型的空间敏感参数满足标度不变性。

因此，若流域水文模型的空间敏感参数与流域面积的关系曲线满足(5)式，则可认为此时流域水文模型的空间敏感参数满足标度不变性。

(5)无资料目标流域的参数确定。对于满足标度不变性的水文模型空间敏感参数，根据步骤(3)中建立的模型参数－面积定量关系，即可将步骤(2)中率定的模型参数转换至无资料的目标流域，用于水文模拟。

2　研究实例

嘉陵江是长江上游的一级支流，发源于秦岭起凤县，途经陕西省、甘肃省、四川省、重庆市，最后注入长江，属于亚热带季风气候，水量充沛，全年水量主要来源于7—9月降雨，占年径流量的50%以上。小河坝流域位于嘉陵江流域西南方向，地势较高，流域面积约28 900 km^2。

本文以嘉陵江小河坝站以上流域为研究区，从干流上游到下游选择平武站以上、江油站以上、涪江桥站以上、射洪站以上、小河坝站以上、平武站－涪江桥站区间、涪江桥站－射洪站区间共7个子流域为嵌套式流域空间；基于新安江三水源模型及2008—2012年日尺度水文气象资料，对前述模型参数空间尺度效应分析方法进行实例应用研究，其中2008—2010年用于模型参数率定，2011—2012年用于模型参数验证。

2.1　新安江模型率定与验证

三水源新安江模型是集总式水文模型，其模型计算的最重要特点是“三分”，即分单元、分水源、分阶段。模型产汇流计算包括以下四部分[14]：(1)蒸散发计算，将土壤纵向分为上层、下层和深层，相应的参数包括KC、WUM、WLM、C；(2)产流计算，采用蓄满产流模式，参数包括WM、B、IMP；(3)水源划分，采用自由水蓄水库将水源分为地表、壤中、地下三种径流成分，参数包括SM、EX、KI、KG；(4)汇流计算，汇流分为坡面汇流、河网汇流两阶段，参数包括CS、CI、CG、KE、XE、L。嘉陵江小河坝站以上流域各子流域三水源新安江模型2008—2010年率定的日模参数见表1，日模率定结果见表2，2011—2012年日模验证结果见表3。

表1　日尺度新安江模型参数率定结果

流域	面积/km^2	KC	B	C	WM	WUM	WLM	IM	SM	EX	KG	KI	CG	CI	CS	L	XE
平武	4 310	0.1	0.2	0.18	120	60	40	0.010	36	1.25	0.15	0.55	0.997	0.96	0.20	0	0.45
江油	5 915	0.5	0.2	0.18	120	60	40	0.010	23	1.25	0.18	0.52	0.995	0.93	0.40	0	0.50
平武-涪江桥	7 593	0.5	0.2	0.18	120	60	40	0.010	20	1.25	0.20	0.50	0.993	0.91	0.32	0	0
涪江桥-射洪	11 671	1.9	0.2	0.18	120	60	40	0.010	14	1.25	0.24	0.46	0.992	0.89	0.23	0	0
涪江桥	11 903	0.4	0.2	0.18	120	60	40	0.010	14	1.25	0.25	0.45	0.992	0.89	0.20	0	0
射洪	23 574	0.7	0.3	0.18	120	60	40	0.030	30	1.30	0.40	0.30	0.994	0.58	0.43	0	0.20
小河坝	28 900	1.0	0.2	0.18	120	60	40	0.015	35	1.25	0.50	0.20	0.995	0.48	0.50	0	0.03

2.2　模型参数空间尺度定量关系分析

已有研究表明[15]，新安江模型中的不敏感参数包括B、C、WM、WUM、WLM、IM、EX，它们主要受到气候条件和下垫面条件的影响，改变后的模拟结果与参数改变前的模拟结果基本不变，反映迟钝，可粗略估计或根据一般经验固定下来，往往不参加优选，因此，将不对其建立参数-面积定量关系，不探究其标度不变性，只研究新安江模型敏感参数KC、SM、KG、KI、CG、CI、CS、L、XE的标度不变性。平武水文站以上流域面积(4 310 km^2)在各子流域中最小，假设平武流域率定出的模型参数为参数“真值”θ_0，其流域面积为F_0；按照前述方法建立新安江模型敏感参数与各子流域面积关系曲线$\theta_i/\theta_0\sim F_i/F_0$，得到模型参数随流域空间尺度变化情况，其中SM、KG、KI、CG、CI、CS这六个参数随流域空间尺度变化最为明显，称之为空间敏感参数，其参数面积关系曲线及其与流域面积间的定量关系见图4。其中SM、CG和CS与流域面积关系的空间尺度范围包括平武子流域(4 310 km^2)、江油子流域(5 915 km^2)、平武-涪江桥子流域(7 593 km^2)、涪

表 2　2008—2010 年日模率定结果

各子流域	年份	总雨量/mm	洪量误差/%	确定性系数
平武	2008	776.0	−0.09	0.71
	2009	797.0	1.58	0.77
	2010	961.1	−1.77	0.89
江油	2008	772.0	−13.88	0.66
	2009	814.0	−9.48	0.84
	2010	939.0	0.32	0.90
平武-涪江桥	2008	811.0	−8.33	0.80
	2009	789.0	−2.20	0.87
	2010	928.0	−1.88	0.91
涪江桥-射洪	2008	927.0	10.22	0.76
	2009	950.0	−12.49	0.81
	2010	915.0	12.18	0.80
涪江桥	2008	798.0	2.77	0.76
	2009	792.0	11.82	0.70
	2010	940.0	4.42	0.87
射洪	2008	927.0	−5.56	0.78
	2009	950.0	0.99	0.87
	2010	915.0	−5.46	0.83
小河坝	2008	926.0	−10.24	0.63
	2009	952.0	0.42	0.78
	2010	931.0	2.16	0.75

表 3　2011—2012 年日模验证结果

各子流域	年份	总雨量/mm	洪量误差/%	确定性系数
平武	2011	981	1.80	0.83
	2012	815	−18.87	0.74
江油	2011	947	9.71	0.69
	2012	814	−3.59	0.50
平武-涪江桥	2011	926	9.74	0.85
	2012	796	−0.47	0.70
涪江桥-射洪	2011	945	15.68	0.76
	2012	830	19.36	0.52
涪江桥	2011	947	18.59	0.68
	2012	802	−1.59	0.66
射洪	2011	945	1.99	0.80
	2012	830	−5.96	0.66
小河坝	2011	902	−1.97	0.75
	2012	911	10.47	0.71

江桥-射洪子流域（11 671 km^2）和涪江桥子流域（11 903 km^2），KG、KI 和 CI 与流域面积关系的空间尺度范围包括平武子流域、江油子流域、平武-涪江桥子流域、涪江桥-射洪子流域、涪江桥子流域、射洪子流域（23 574 km^2）和小河坝子流域（28 900 km^2）。

由图 3 可知，新安江模型的空间敏感参数与流域面积间定量关系基本满足幂函数形式，即公式（6）对模型参数流域面积定量关系的假设是成立的。因此，新安江模型的空间敏感参数有着随空间尺度变化的标度不变性，幂函数的指数即为标度指数。

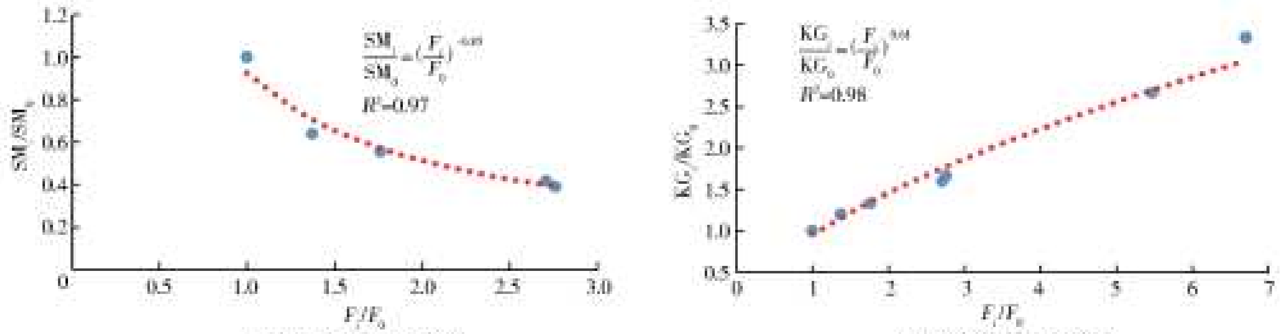
在新安江模型空间敏感参数中，SM、CG、CS、KG、KI、CI 在其相应的空间尺度范围内，参数与流域面积间存在较好的幂函数关系，这主要是由于分形理论的标度不变性有一定的适应范围，即标度区间。因此，SM、CG、CS、KG、KI、CI 在其相应的标度区间内存在标度不变性。同时，新安江模型参数 KG 在标度区间内随流域面积增大呈现上升趋势，而 SM、KI、CG、CI、CS 在标度区间内随流域面积增大呈现下降趋势。

综上可知，对具有标度不变性的参数，推求其标度指数后，在相应的标度区间内，即可实现模型参数在不同空间尺度上的转换，利用在有资料流域率定的参数去推求无资料流域的模型参数。

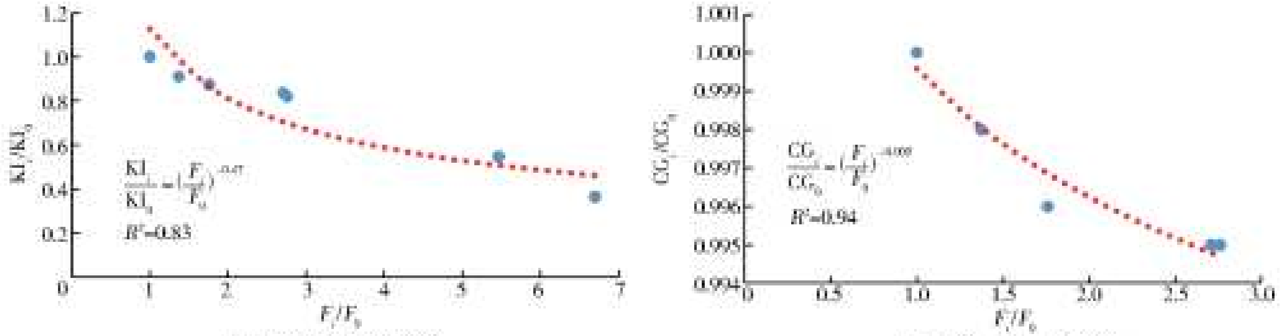
2.3　参数空间尺度效应的检验

根据前述建立的新安江模型参数与流域面积的标度关系，假设小河坝天仙寺流域是无资料地区，其流域面积为4 976 km^2，现要模拟其 2008—2011 年逐日流量过程。利用新安江模型空间敏感参数的标度不变性及其与流域面积定量关系，推求得到天仙寺流域相应模型参数值 SM、KG、KI、CG、CI、CS，其他需要的新安江模型参数有 KC、B、C、WM、WUM、WLM、IM、EX、L、XE，由于射洪子流域与天仙寺流域相近，根据水文相似性，移用射洪子流域相应模型参数值，各模型参数值见表 4，模拟结果见表 5，流量模拟过程线见图 4。

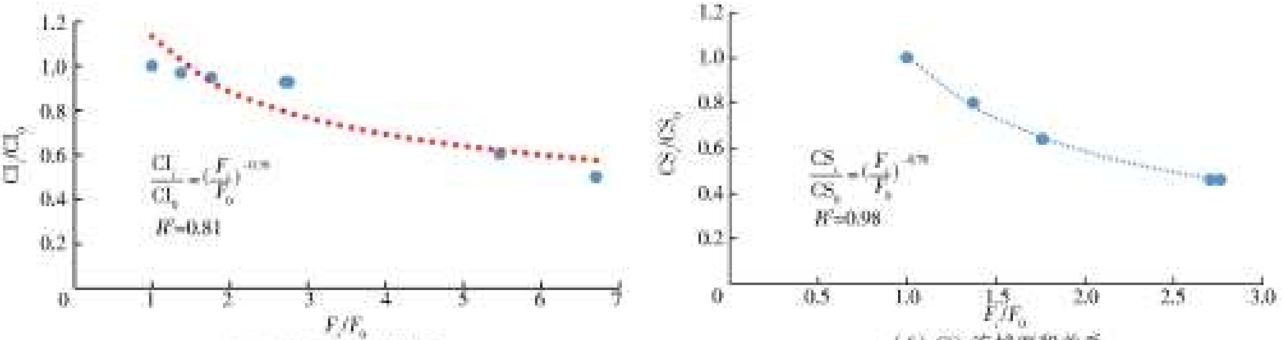
结果表明，基于分形理论利用标度变换所得模型参数模拟天仙寺流域 2008—2011 年流量过程，平均确定性系数为 0.65，洪量相对误差在 3%以内，效果较好。因此，在标度区间内，已知新安江空间敏感参数的标度指数，参证站的空间敏感参数值和无资料地区流域面积，便可利用标度变化得到无资料地区模型参数的估计值，进行水文模拟，实现有资料地区水文模型参数值向无资料地区的移用，其中模型参数的标度指数可以通过绘制参数流域面积关系图进行推求。



（a）SM-流域面积关系　　（b）KG-流域面积关系



（c）KI-流域面积关系　　（d）CG-流域面积关系



（e）CI-流域面积关系　　（f）CS-流域面积关系

图3　新安江模型参数—流域面积关系

表4　天仙寺流域模型参数

模型参数	SM	KG	KI	CG	CI	CS	KC	B
参数值	32	0.16	0.54	0.996	0.913	0.40	0.7	0.3
模型参数	C	WM	WUM	WLM	IM	EX	L	XE
参数值	0.18	120	60	20	0.02	1.25	0	0.03

表5　天仙寺流域日模模拟结果

年份	2008	2009	2010	2011	2008—2011
总雨量/mm	1 015	1 102	932	999	7 163
洪量误差	−2.31	−2.70	−0.90	−0.10	2.18
确定性系数	0.59	0.58	0.73	0.65	0.65

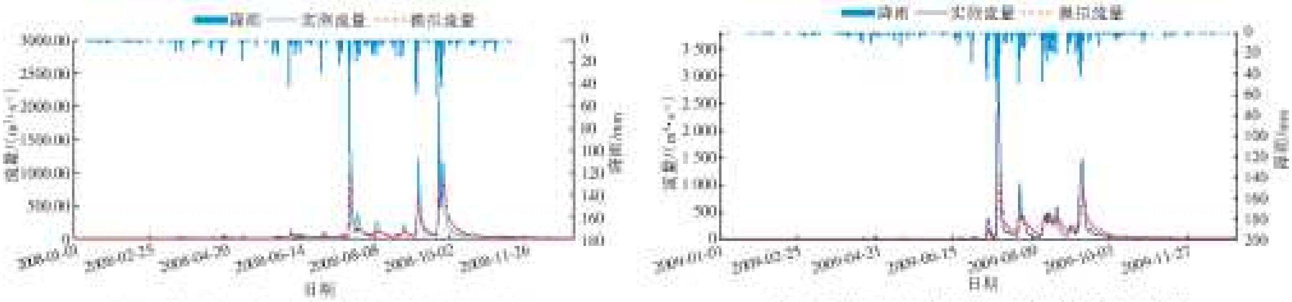
3　结　论

基于分形理论，本文提出一种水文模型参数空间尺度效应分析方法，建立不同空间尺度之间参数定量转换方程。选择三水源新安江模型，在嘉陵江小河坝站以上流域进行应用检验，得到如下主要结论。

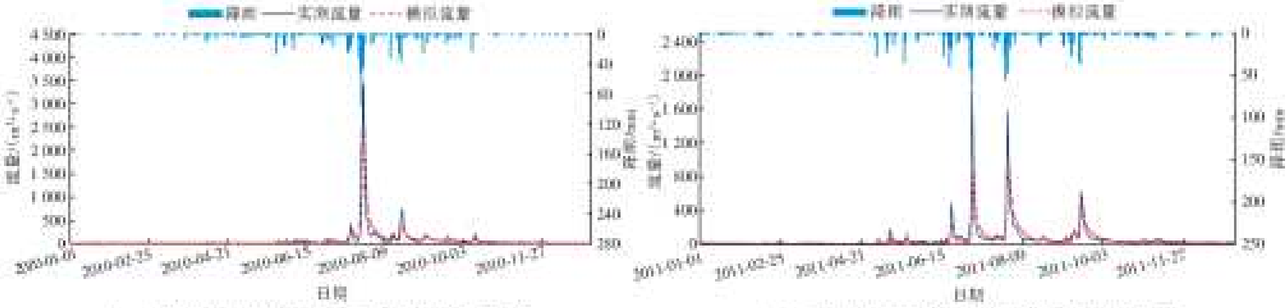
（1）新安江模型中的空间敏感参数SM、KG、KI、CG、CI、CS具有随空间尺度变化的标度不变性，与流域面积间定量关系均可用幂函数形式表示。

（2）在标度区间内，可利用标度变换对模型空间敏感参数进行处理，实现有资料地区水文模型参数值向无资料地区的信息转换，可为无资料地区水文预报提供支撑。

虽然本文研究是以新安江模型为例开展的参数空间尺度效应分析，但是方法思路亦可用于分析其他水文模型参数的空间尺度效应。



(a) 天仙寺2008年日模实测和模拟流量过程　(b) 天仙寺2009年日模实测和模拟流量过程



(c) 天仙寺2010年日模实测和模拟流量过程　(d) 天仙寺2011年日模实测和模拟流量过程

图 4　天仙寺 2008—2011 年日模型实测和模拟流量过程

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·译文(Translation)·

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Spatial scale effect analysis of Xin′anjiang model parameters based on fractal theory

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Abstract: Hydrological scale effect has always been the frontier research in hydrology. In this paper, a quantitative conversion equation between different spatial scales of a model′s sensitive parameters is established based on the fractal theory, and a method for analyzing the spatial scale effects of hydrological model parameters is proposed. Taking the three-source Xin′anjiang model as an example, the spatial scale effect of model parameters is analyzed above the Xiaoheba station of Jialing River basin. The results showed that the spatial sensitive parameters (SM, KG, KI, CG, CI, and CS) of the Xin′anjiang model are scale-invariant with the change of spatial scale, and the quantitative relationship between them and catchment area can be described by power function relationship. Therefore, the corresponding quantitative conversion equation can be established by this relationship. Through the application and validation on different spatial scales in the basin, it is found that this spatial scale conversion equation of the Xin′anjiang model based on the scale-invariance using fractal theory has high applicability. It can realize that the migration of model parameters can be done from the data-rich region to the no data region and can provide references for the determination of model parameters in no data regions.

Key words: hydrological spatial scale; Xin′anjiang model; fractal theory; Xiaoheba basin

Hydrological phenomena may show different characteristics on different temporal and spatial scales, but there are often uniform laws behind them. Strengthening research on hydrological scale effect and establishing the transformation relationship between different scales can not only deepen the understanding of the hydrological process but also solve the problem of hydrological information transfer from the regions with data to the regions without data and between different time and space scales[1]. Therefore, the study of hydrological scale has important theoretical significance and practical value and has always been the preface of hydrological science research[2-4].

Early research on hydrological scale problems mainly used statistical correlation (regression) methods, such as the empirical formula for flood peaks which are commonly used at home and abroad[5]. In recent years, new theories and methods such as chaos, fractal, and wavelet analysis have been introduced for the research and application of hydrological scale problems. Sivakumar[6] applies chaos theory for the first time to study the scale effect of rainfall. The results showed that the change in the weight of rainfall data at different time scales is chaotic, and proposed a chaos model to solve the problem of rainfall downscaling. Kumar[7-8] integrates the fractal theory and wavelet

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Author brief: LI Binquan (1984-), male, professor, Ph. D., mainly engaged in the research on hydrology and water resources. E-mail: libinquan@hhu.edu.cn analysis and proposed a method to separate the large-scale and small-scale features of the spatial rainfall field, and documented that rainfall fluctuations can be approximated as a stable distribution. Gupta et al.[9] introduce the scale invariance hypothesis for regional flood analysis, which provide a basis for studying the spatial variability of hydrological processes at different scales. In China, Wang Wensheng et al.[10] use the time-frequency localization function of wavelet transform to show the frequency characteristics of hydrological series in the time domain and obtained the strength and distribution of various periods as well as the mutation points. Li Xianbin[11] uses the fractal and wavelet coupling method to carry out the daily runoff forecast in the flood season and reveales that the results are better than the differential analysis method and fractal dimension estimation method. Chang Fuxuan et al.[12] use the fractal theory to analyze the scale between the flood peaks and the area of the Jialing River basin and found that the changes of the flood peaks with the area of the basin are scale-invariant.

Most of the previous studies focus on the scale problems of hydrological variables (such as rainfall, peak discharge, etc.), but there are few studies on the scale effects of hydrological model parameters. This paper selects the basins above the Xiaoheba station on the Jialing River and studies the spatial scale invariance of the Xin′anjiang model parameters based on the fractal theory in order to reveal the spatial scale effects of the model parameters and to establish the conversion relationship between the model parameters at different spatial scales and to solve the problem of hydrological model parameters of an area that has no data.

1 Research methods

1.1 Fractal theory and its scale invariance

Bloschl and Sivapalan[1] have proposed two methods to study the scale problem, one is the model-based approach, the other is the system integrity approach based on the dimension analysis and similarity, which mainly involves the concept of dimension analysis and fractal. Fractal is widely used in hydrological scale analysis because it can easily handle complex processes.

The fractal theory is founded by Mandelbrot in the mid-1870s, and refers to the parts that make up the whole, whose shape is similar to the whole in some way, with self-similarity and scale invariance[13]. Mathematically, self-similarity in fractal theory means scale invariance. For scale-invariant research objects, within a certain scale range, the relationship between certain attributes of different scales can be determined by the scale transformation only related to the scale ratio. This scale range is called the scale interval. Beyond the scale interval, the variable may no longer satisfy scale invariance. Therefore, if a certain hydrological variable satisfies the scale invariance, the hydrological characteristics (or laws) at different scales can be derived through scaling transformation.

Define a random field $\{Y(x)|x\in R^d\}$, where x is time, one-dimensional or multidimensional space or space-time, and d is the dimension of this space. For any constant $\lambda>0$, define $Y_\lambda(x)=Y(\lambda x)$. If (λx) and $\lambda^\alpha Y(x)$ have the same probability distribution function, that is

$$\{Y(\lambda x)\}\underline{\underline{dis}}\lambda^\alpha\{Y(x)\} \tag{1}$$

Then, it is said that $Y(x)$ satisfies the scale invariance or has the scale property, and the index α is called the scale index[9]. Equation (1) indicates that for any probability P, there are

$$Y(\lambda x)=\lambda^\alpha Y(x) \tag{2}$$

or

$$\log(Y(\lambda x))=\alpha\log\lambda+\log Y(x) \tag{3}$$

Scale invariance is an important feature of fractals. This article uses fractal theory to select sub-catchments of different sizes from the basin with similar geographical and climatic conditions (or hydrological consistency zones) to analyze the spatial scale effect of the hydrological model parameters to determine whether hydrological model parameters are scale-invariant with changes in spatial scale or not. For model parameters with scale invariance, the parameters can be converted between different spatial scales.

1.2　Model parameter invariance analysis

In practical applications, the scale invariance analysis results of certain hydrological model parameters can be established on a number of different spatial scales and hydrological similarity, and the calibrated model parameters of a basin with data can be transferred to the basin without data. The main process steps are as follows;

(1) Select nested hydrological similarity contrast basins. For a targeted basin with no data, several comparative basins with spatial nesting and hydrological similarity are selected, as shown in Fig. 1.

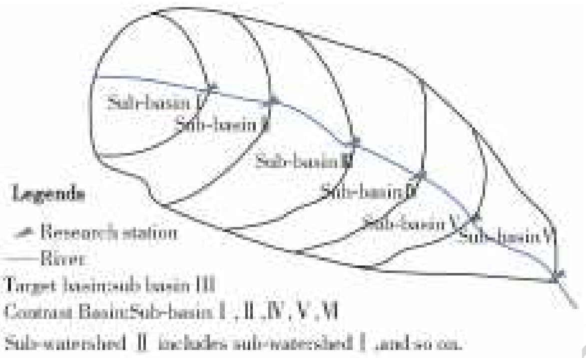


Fig. 1　Spatial basin diagram

(2) Calibration of hydrological model parameters. In several spatially nested basins, hydrological models are constructed using meteorological and hydrological data, and model parameters in each basin are determined.

(3) Establish a quantitative relationship between model parameters and the basin area. Take the minimum sub-basin model parameter value θ_0 as the true value, corresponding to the basin area F_0; each sub-basin model parameter value θ_i, corresponding to the basin area F_i. Take the ratio of each sub-basin model parameter θ_i to the true value θ_0, that is, θ_i/θ_0 as the ordinate, and the corresponding basin area F_i/F_0 as the abscissa, and establish the relationship between the model parameter and the basin area $\theta_i/\theta_0 \sim F_i/F_0$ (Fig. 2). The spatially sensitive parameters of the hydrological model are selected according to the relationship curve, and the quantitative relationship between the two is established using mathematical-statistical methods $\theta_i/\theta_0 = f(F_i/F_0)$.

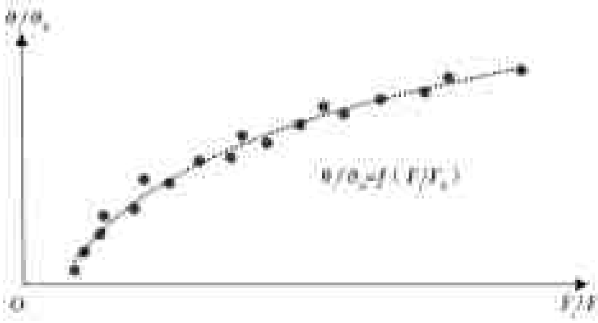


Fig. 2　Relationship curve between model parameters and watershed area

(4) Space scale invariance test of model parameters. Let $\theta(F_j)$ be a sensitive model parameter that varies with the spatial scale on a basin area F_j. It is affected by a variety of factors and has a large uncertainty. Assume it is a random variable, and let the scale F_i be λ times F_j

$$F_i=\lambda F_j \tag{4}$$

Similar to $\theta(F_j)$, $\theta(F_i)$ is used as a random variable corresponding to the scale F_i. Suppose that;

$$\theta(F_i)=\lambda^\alpha\theta(F_j) \tag{5}$$

where α is a constant. According to the relevant knowledge of the distribution density function of random variables in statistics, let $\theta(F_i)$ distribution density function be $f_i(\theta_i)$ and $\theta(F_j)$ distribution density function be $f_j(\theta_j)$. Then

$$f_i(\theta_i)=f_j(g^{-1}(\theta(F_i)))\times\frac{dg^{-1}(\theta(F_j))}{d\theta(F_j)}=$$

$$f_j\left(\frac{\theta(F_i)}{\lambda^\alpha}\right)\times\lambda^\alpha \tag{6}$$

Exit

$$\theta(F_i)\underline{\underline{dis}}\lambda^\alpha\theta(F_j) \tag{7}$$

According to the basic formula (1) of scale invariance, the spatial sensitive parameters of basin model can meet the scale invariance.

Therefore, if the relationship between the spatially sensitive parameters of the basin hydrological model and the area of the basin satisfies Equation (5), it can be considered that the spatially sensitive parameters of the basin hydrological model meet the scale invariance at this time.

(5) Parameter determination of target basin without data. According to the quantitative relationship between model parameters and area estab-

lished in step (3), the model parameters calibrated in step (2) can be converted to the target basin that has no data for hydrological simulation.

2 Case study

The Jialing River is a tributary of the upper reaches of the Yangtze River. It originated from Qifeng County, Qinling mountains, passes through Shaanxi Province, Gansu Province, Sichuan Province, and Chongqing City, and finally flows into the Yangtze River. It has a subtropical monsoon climate with abundant water. The annual water volume mainly comes from July to September rainfall, accounting for more than 50% of the annual runoff. The Xiaoheba basin is located in the southwest of the Jialing River basin, and its terrain is relatively high. The basin area is about 28 900 km^2.

In this paper, the watershed above the Xiaoheba station of Jialing River is taken as the research area. From the upstream to the downstream of the mainstream, seven subwatersheds are selected as nested watershed spaces: above Pingwu station, above Jiangyou station, above Fujianqiao stations, above Shehong station, above Xiaoheba station, between Pingwu station and Fujianqiao station, and among Fujianqiao station and Shehong station, respectively. Based on the Xin′anjiang (three water source) model and the daily scale hydrometeorological data from 2008 to 2012, the spatial scale effect analysis method of the above model parameters is applied to the case study, in which the model parameters are calibrated from 2008 to 2010 and validated from 2011 to 2012.

2.1 Calibration and validation of Xin′anjiang Model

The three-water source Xin′anjiang model is a lumped hydrological model, and the most important feature of its model calculation is "three parts", namely, unit, water source, and stage. The calculation process to generate model runoff includes the following four parts[14]: (1) evapotranspiration calculation: which divides the soil vertically into upper, lower and deep layers, and the corresponding parameters include KC, WUM, WLM, C. (2) runoff calculation: using full-storage runoff mode, parameters include WM, B, abd IMP. (3) water source division: using free water storage reservoirs to divide water sources into three types of runoff components: surface, middle and underground; parameters include SM, EX, KI, and KG. (4) convergence calculation: Convergence is divided into two phases: slope convergence and river network convergence. The parameters include CS, CI, CG, KE, XE, and L. The daily model parameters for the three-source Xin′anjiang model from 2008-2010 in the sub-basin above the Xiaolingba Station on the Jialing River are shown in Tab. 1. The daily model calibration results are shown in Tab. 2, while the daily model validation results are presented in Tab. 3.

Tab. 1 Parameters calibration results of daily-scale Xin′anjiang model

basin/Watershed	Area/km^2	KC	B	C	WM	WUM	WLM	IM	SM	EX	KG	KI	CG	CI	CS	L	XE
Pingwu	4 310	0.1	0.2	0.18	120	60	40	0.010	36	1.25	0.15	0.55	0.997	0.96	0.20	0	0.45
Jiangyou	5 915	0.5	0.2	0.18	120	60	40	0.010	23	1.25	0.18	0.52	0.995	0.93	0.40	0	0.50
Pingwu-Fujiangqiao	7 593	0.5	0.2	0.18	120	60	40	0.010	20	1.25	0.20	0.50	0.993	0.91	0.32	0	0
Fujiangqiao-Shehong	11 671	1.9	0.2	0.18	120	60	40	0.010	14	1.25	0.24	0.46	0.992	0.89	0.23	0	0
Fujiangqiao	11 903	0.4	0.2	0.18	120	60	40	0.010	14	1.25	0.25	0.45	0.992	0.89	0.20	0	0
Shehong	23 574	0.7	0.3	0.18	120	60	40	0.030	30	1.30	0.40	0.30	0.994	0.58	0.43	0	0.20
Xiaoheba	28 900	1.0	0.2	0.18	120	60	40	0.015	35	1.25	0.50	0.20	0.995	0.48	0.50	0	0.03

2.2 Quantitative relationship analysis for model parameters at a spatial scale

Previous studies have shown that the insensitive parameters in the Xin′anjiang model include B, C, WM, WUM, WLM, IM, and EX, are mainly affected by climatic conditions and underlying surface conditions[15]. The simulation results before and after the parameter change are basically the same, which are slow to reflect, can be roughly estimated or fixed according to general experience , and often do

Tab. 2　Calibration results of the daily model from 2008 to 2010

Sub-basin	Year	Total rainfall/mm	Flood volume error/%	Deterministic coefficient coefficient
Pingwu	2008	776.0	−0.09	0.71
	2009	797.0	1.58	0.77
	2010	961.1	−1.77	0.89
Jiangyou	2008	772.0	−13.88	0.66
	2009	814.0	−9.48	0.84
	2010	939.0	0.32	0.90
Pingwu-Fujiangqiao	2008	811.0	−8.33	0.80
	2009	789.0	−2.20	0.87
	2010	928.0	−1.88	0.91
Fujiangqiao-Shehong	2008	927.0	10.22	0.76
	2009	950.0	−12.49	0.81
	2010	915.0	12.18	0.80
Fujiangqiao	2008	798.0	2.77	0.76
	2009	792.0	11.82	0.70
	2010	940.0	4.42	0.87
Shehong	2008	927.0	−5.56	0.78
	2009	950.0	0.99	0.87
	2010	915.0	−5.46	0.83
Xiaoheba	2008	926.0	−10.24	0.63
	2009	952.0	0.42	0.78
	2010	931.0	2.16	0.75

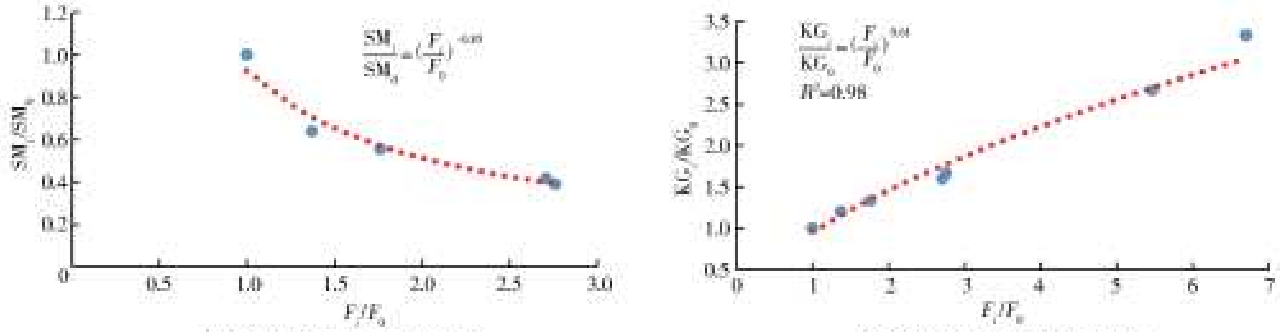
Tab. 3　Validation results of the daily model from 2011 to 2012

Sub-basin	Year	Total rainfall /mm	Flood volume error/%	Deterministic coefficient coefficient
Pingwu	2011	981	1.80	0.83
	2012	815	−18.87	0.74
Jiangyou	2011	947	9.71	0.69
	2012	814	−3.59	0.50
Pingwu-Fujiangqiao	2011	926	9.74	0.85
	2012	796	−0.47	0.70
Fujiangqiao-Shehong	2011	945	15.68	0.76
	2012	830	19.36	0.52
Fujiangqiao	2011	947	18.59	0.68
	2012	802	−1.59	0.66
Shehong	2011	945	1.99	0.80
	2012	830	−5.96	0.66
Xioaheba	2011	902	−1.97	0.75
	2012	911	10.47	0.71

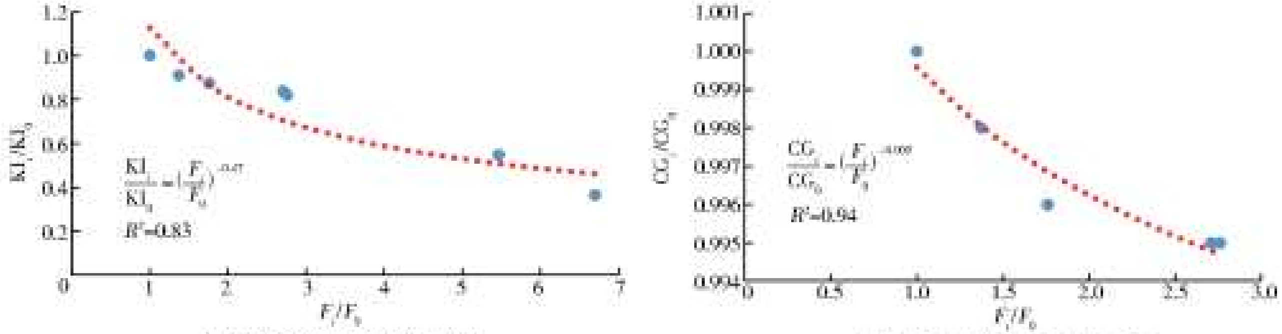
not participate in optimization. Therefore, they will not establish a quantitative relationship between parameters and area and do not explore the invariance of their scale. Hence, the scale invariance of the sensitive parameters KC, SM, KG, KI, CG, CI, CS, L, and XE of the Xin′anjiang model is studied. The basin area above Pingwu hydrological station (4 310 km^2) is the smallest compared to other sub-basin. It is assumed that the model parameter determined by the Pingwu basin rate is the parameter "true value" θ_0, and its basin area is F_0. The Xin′anjiang model is established according to the aforementioned method. The relationship curve between sensitive parameters and the area of each sub-basin is $\theta_i/\theta_0 \sim F_i/F_0$, the model parameters change with the spatial scale of the basin. Among them, the six parameters such as SM, KG, KI, CG, CI, and CS change significantly with the spatial scale of the basin. It is called a spatially sensitive parameter, and its parameter area relationship curve and its quantitative relationship with basin area are shown in Fig. 4. The spatial scale of the relationship between SM, CG, and CS and basin area includes the Pingwu sub-basin (4 310 km^2), the Jiangyou sub-basin (5 915 km^2), the Pingwu-Qujiang sub-basin (7 593 km^2), and the Fujianqiao Shehong sub-basin (11 671 km^2) and Fujianqiao sub-basin (11 903 km^2), respectively. The spatial scale relationship between KG, KI, and CI and the area of the basin comprise Pingwu sub-basin, Jiangyou sub-basin, and Pingwu-Qujiang Bridge sub-basin, Qijiangqiao-Shehong sub-basin, Fujianqiao sub-basin, Shehong sub-basin (23 574 km^2) and Xiaoheba sub-basin (28 900 km^2), respectively.

It can be seen from Fig. 3 that the quantitative relationship between the spatially sensitive parameters of the Xin′anjiang model and the basin area meets the power function form, therefore, the assumption of formula (6) on the quantitative relationship between the basin area of the model parameters is tenable. Hence, the spatially sensitive parameters of the Xin′anjiang model have scale invariance with the change of spatial scale, and the exponent of the power function is the scale index.

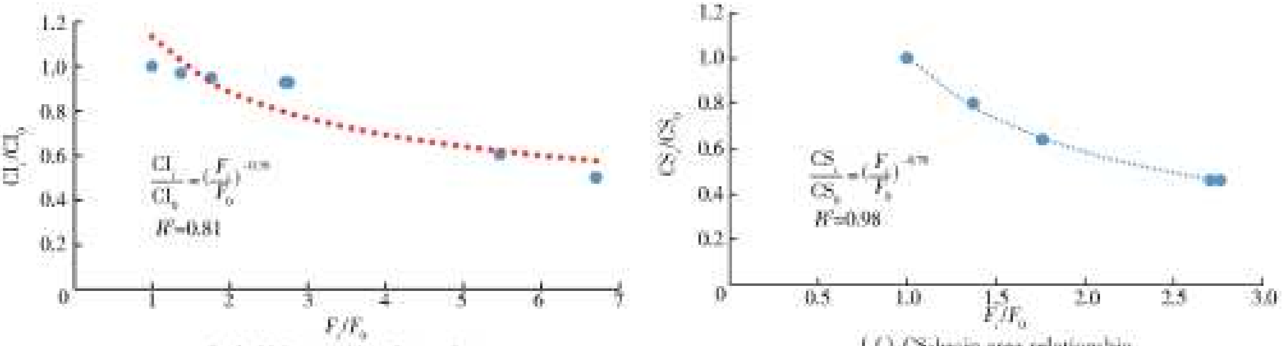
Among the spatially sensitive parameters of the Xin′anjiang model, SM, CG, CS, KG, KI, CI are within their corresponding spatial scales, and there is a good power function relationship between the parameters and the area of the basin. The scale invariance of fractal theory has a certain adaptation range, namely scale range. Therefore, SM, CG, CS, KG, KI, and CI have scale invariance in their corresponding scale intervals. Simultaneously, the Xin′anjiang model parameter KG showed an upward trend with the increase of the basin area in the scale interval, while SM, KI, CG, CI, and CS showed a downward trend with the increase of the basin area in the scale interval.



(a) SM-basin area relationship (b) KG-basin area relationship



(c) KI-basin area relationship (d) CG-basin area relationship



(e) CI-basin area relationship (f) CS-basin area relationship

Fig. 3 Diagram for the relationship between model parameter-basin area

In summary, for the parameters with scale invariance, the conversion of model parameters on different spatial scales can be realized in the corresponding scale interval after the scale index is calculated, and the model parameters of the basins that have no data can be calculated by the calibrated parameters in the basins that has specific data.

2.3 Test of parameter′s spatial scale effect

According to the scale relationship between the parameters of the Xin′anjiang model and the basin area, it is assumed that the Xiaoheba Tianxiansi basin with a drainage area (4 976 km^2) has no data. Therefore, it is necessary to simulate its daily flow from 2008 to 2011. Based on the scale invariance of spatial sensitive parameters of the Xin′anjiang model and its quantitative relationship with basin area, the corresponding model parameters of the Tianxiansi basin such as SM, KG, KI, CG, CI, CS, are obtained. Other required parameters of the Xin′anjiang model are KC, B, C, WM, WUM, WLM, IM, EX, L, and XE, respectively. The Shehong sub-basin is similar to the Tianxiansi basin with respect to the hydrological similarity, the corresponding model parameter values of the Shehong sub-basin are transferred, and for each model parameter value (see Tab. 4), simulation results in Tab. 5, and flow simulation process line in Fig. 4.

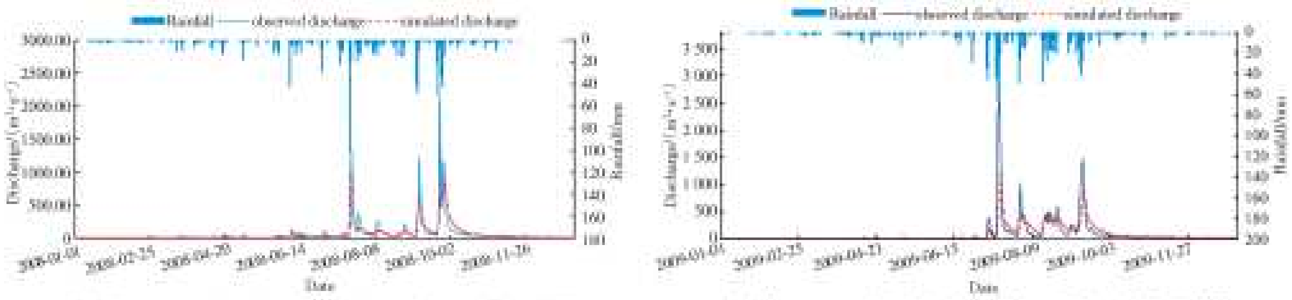
The results based on fractal theory show that the model parameters obtained by scaling transformation are used to simulate the flow process of the Tianxiansi basin from 2008 to 2011. The average deterministic coefficient is 0.65, and the relative error of flood volume is within 3%, respectively. The effect is good. Therefore, in the scale interval, if the scale index of the spatially sensitive parameters of Xin′anjiang River is known, the values of the spatially sensitive parameters of the reference station and the area of the basin with no data can be used to obtain the estimated values of the model parameters of the sparse data region. Hydrological simulation is performed to realize the transfer of hydrological model parameter values in areas with data to areas without data. The scaled index of model parameters can be derived by drawing a parameter basin area relationship diagram.

Tab. 4 Parameters of Tianxiansi basin

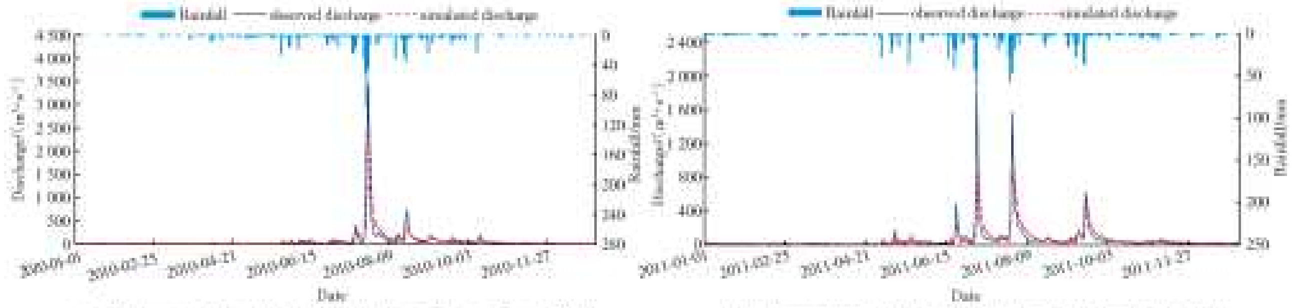
Model parameter	SM	KG	KI	CG	CI	CS	KC	B
Parameter value	32	0.16	0.54	0.996	0.913	0.40	0.7	0.3
Model parameter	C	WM	WUM	WLM	IM	EX	L	XE
Parameter value	0.18	120	60	20	0.02	1.25	0	0.03

Tab. 5 Daily model simulation results of Tianxiansi basin

Year	2008	2009	2010	2011	2008-2011
Total rainfall/mm	1 015	1 102	932	999	7 163
Flood volume error	−2.31	−2.7	−0.9	−0.1	2.18
deterministic coefficient	0.59	0.58	0.73	0.65	0.65



(a) Measurement and simulation of daily discharge in Tianxiansi Basin in 2008 (b) Measurement and simulation of daily discharge in Tianxiansi Basin in 2009



(c) Measurement and simulation of daily discharge in Tianxiansi Basin in 2010 (d) Measurement and simulation of daily discharge in Tianxiansi Basin in 2011

Fig. 4 Measured and simulated flow process of Tianxiansi basin from 2008 to 2011

3 Conclusions

This paper proposes a method for analyzing the spatial scale effect of hydrological model parameters and establishes quantitative conversion equations of parameters between different spatial scales based on the fractal theory. The three-source Xinanjiang model was selected and applied to the basin above the Xiaoheba station on the Jialing River to obtain the following conclusions:

(1) The spatially sensitive parameters SM, KG, KI, CG, CI, and CS in the Xin′anjiang model have scale invariance that varies with the spatial scale, and the quantitative relationship with the basin area can be expressed in the form of a power function.

(2) In the scale interval, the scale transformation can be used to process the spatial sensitive pa-

rameters of the model to realize the conversion of the hydrological model parameter values in the area with data to the area without data and provide support for the hydrological forecast in the area that has no data.

Although this study uses the Xin′anjiang model as an example to analyze the spatial scale effect of parameters, the method and the idea can also be used to analyze the spatial scale effect of other hydrological model parameters.

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