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基于区间犹豫模糊语言集的水资源多目标决策

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摘要:以鄂北水资源配置工程为对象,研究大型调水工程背景下的多水源多目标优化配置。将鄂北地区水库分为充蓄水库、补偿水库与在线水库等3类,根据各类水库特点对鄂北水资源配置工程进行系统概化,建立以年缺水量最小和调水成本最低为目标函数的多水源多目标优化配置模型。采用权重系数法进行求解得到非劣解集。最后采用基于区间犹豫模糊语言与TOPSIS的多目标决策方法对各方案的缺水量与调水量作出评价并遴选出最优方案。研究结果表明:对鄂北水资源配置工程而言,缺水量与调水量的权重比为7∶3的方案综合评价最优,重视缺水的同时兼顾调水成本;基于区间犹豫模糊语言集与TOPSIS的水资源多目标决策方法经本文实例验证具有较高的稳定性与可行性。

关键词:调水工程;优化配置;多目标;犹豫模糊语言;TOPSIS

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水资源优化配置是水资源合理高效利用的重要措施,也是当今水资源领域研究的重点问题[1-3]。国内外对水资源优化配置的研究[4-5]已有较大进展,从单目标转向了多目标,即从单一地追求经济目标转向综合考虑社会、经济、环境等多目标,从单水源的优化调度转向多水源联合优化调度的复杂配置。张翔宇等[6]建立了黄河干流河段多目标优化配水模型,采用改进遗传算法求解河段优化配置问题;闫堃等[7]以加权相对总缺水深度最小和系统供水总成本最小为目标建立了多目标优化模型,并采用多目标粒子群算法(MOPSO)求解;顾文权等[8]建立了多目标风险分析模型,提出了基于随机模拟技术的水资源优化配置多目标风险评估方法;付强等[9]在构建灌区水资源供需联合分布对其耦合量化模型的基础上分析水资源短缺遭遇概率后构建区间线性规划模型;高黎明等[10]根据区域用水需求与总量控制及水功能区限制纳污管控要求,建立基于水量水质双控的区域水资源优化配置模型,采用AHP法、熵权法、AHP-熵权耦合法计算不同情景方案的配置效益并优选配置方案。但从已有文献看,多目标决策方法也尚在发展之中。本文以包含大型调水工程的区域为背景,构建以年缺水量最小和调水成本最小为目标的多水源多目标优化配置模型,采用权重法求解优化配置模型得到非劣解集,采用区间犹豫模糊语言集与TOPSIS相结合的多目标决策方法对方案集进行优选,以鄂北水资源配置工程为对象开展实例研究并推荐最佳水量调度方案。

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1　水资源配置工程概化

鄂北地区水资源配置工程利用清泉沟隧洞从丹江口水库引水,在不影响南水北调中线工程调水规模和过程的前提下,解决鄂北地区水资源短缺问题,满足鄂北受水区生活、生产以及生态用水需求。

鄂北水资源优化配置模型较为复杂,包含多水源、多用户与多节点,元素概化主要分为节点、水库和用户等 3 种形式。节点分为入库节点、退水节点、干渠控制节点和分水口分水节点;水库分为充蓄水库、补偿水库(虚拟水库,在总干渠分水口为直供和补偿的情况下设置)和在线水库;用户分为生活用水、生态用水、工业用水和农业用水。

1.1　充蓄调节水库

充蓄调节水库是指该水库会接收分水口的外引水,并且这部分分水会直接进入充蓄水库中,与天然来水一起进行调节分配,再供给各个用户。具体概化见图 1。

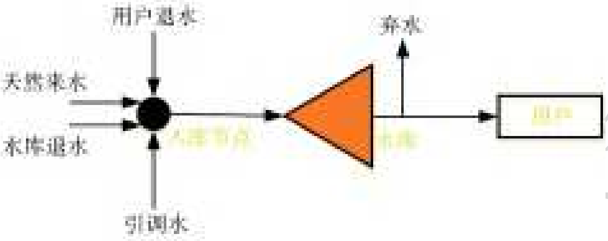

图 1　充蓄水库模型概化

水库的水量平衡方程为

$$V_{i,t+1} = V_{i,t} + W_{R_{i,t}} + W_{K_{i,t}} + W_{Y_{i,t}} + W_{T_{i,t}} - O_{i,t} - S_{S_{i,t}} \qquad (1)$$

式中:$V_{i,t+1}$和 $V_{i,t}$ 分别表示第 i 水库第 $t+1$ 和第 t 时刻的蓄量;$W_{R_{i,t}}$ 表示第 i 水库第 t 时段的天然来水;$W_{K_{i,t}}$ 表示第 i 水库第 t 时段接收的其他水库退水;$W_{Y_{i,t}}$ 表示第 i 水库第 t 时段接收的工程引调水;$W_{T_{i,t}}$ 表示第 i 水库第 t 时段接收的用户退水;$O_{i,t}$ 表示第 i 水库第 t 时段的弃水量;$S_{S_{i,t}}$ 表示第 i 水库第 t 时段的供水量。

此外,在对充蓄水库进行蓄量限制时,要在蓄量的基础上加上引水量再对蓄量进行进一步的限制,应满足如下蓄量约束

$$V_{\max_i} \quad V_{i,t+1} + W_{YK_{i,t}} \qquad (2)$$

式中:$W_{YK_{i,t}}$ 表示第 t 时段第 i 水库的引水量。

1.2　补偿调节水库

补偿水库是指该水库会接收分水口的外引水,但是这部分外引水不能自流入库进行调节,而是引至补偿水库的下游渠道,这部分水与水库调蓄的本地径流一同为下游用户供水。因此,补偿水库主要通过对本地径流的重新调节实施对引水的补偿。在建模过程中,补偿水库下游建立一个库容为 0 的虚拟水库(即引水通道),分水口的引水不进入补偿水库而经过虚拟水库直接供给对应用户,即该区域的用户由补偿水库与对应的虚拟水库一起供水。具体模型概化见图 2。

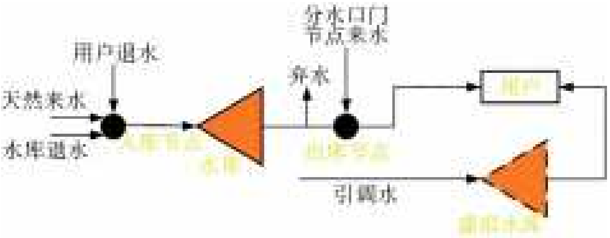

图 2　补偿水库模型概化

1.3　在线水库(封江口水库)

在线水库是指该水库处于干渠渠道之上,所有干渠水与自身天然来水都会直接入库进行综合调蓄后由在线水库供给对应用户,其他出库水(包括弃水)会沿干渠继续流至下游河道。因此,在线水库作为中转站对干渠水的拦蓄作用相当重要。具体模型概化见图 3。

图 3　在线水库模型概化

本区域水资源配置工程概化为元素与关系两部分。经过概化,最终元素有 79 个节点、196 个水库、11 个虚拟水库以及 148 个用户。

2　多水源多目标优化配置模型

鄂北水资源配置工程利用清泉沟隧洞从丹江口水库引水,分别进入引丹总干渠和鄂北引水干渠两条线路。引丹总干渠供给唐西地区各水库,再由各水库供给各个片区的用户;鄂北引水干渠通过分水口门供给鄂北地区沿线水库,再由各水库供给各个片区的用户。鄂北水资源配置系统是相当复杂的大系统,其中需要协调 3 方面关系:一是本地水与外调水的关系,重点考虑调水成本,确定调水时机和调水在各受水区的分配;二是各类水库相互补偿关系,确定各水库的最优供水次序以提升当地水资源的利用率;三是水库供水与需水过程的关系,利用水库调蓄,使供水过程尽可能满足需水过程的要求。对于

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如此复杂的多水源、多目标、多用户配置问题, 首要任务是构建一个整体的大系统多目标优化模型, 通过优化区域中的调水工程运用方式, 优先利用本地水资源, 减少调水量, 以期实现缺水量最小和调水成本最低的双重目标。

2.1 目标函数

考虑水资源优化配置的社会和经济两个目标, 以年总缺水量最小反映社会目标, 以工程调水的成本最低反映经济目标。

以年总缺水量最小为目标

$$F_1 = \min \sum_{j=1}^{J}\sum_{t=1}^{T} a_j \cdot (D_{j,t} - S_{j,t}) \tag{3}$$

式中: a_j 为第 j 个用户的权重系数, 表示每个用户的重要程度; $D_{j,t}$ 表示第 j 个用户第 t 时段的需水量, 万 m^3; $S_{j,t}$ 表示第 j 个用户第 t 时段接收的供水量, 万 m^3。

以工程调水成本最低为目标

$$F_2 = \min \beta \sum_{i=1}^{I}\sum_{t=1}^{T} Y_{i,t} \tag{4}$$

式中: $Y_{i,t}$ 表示第 i 个水库第 t 时段的调水量, 万 m^3; β 为调水成本, 万元/万 m^3。

采用权重法, 将多目标优化问题转化为单目标优化问题

$$F = \min[\lambda F_1 + (1-\lambda)F_2] \tag{5}$$

式中: λ 为缺水量的权重系数, 表示缺水量相对于调水量的重要程度。

2.2 约束条件

水库蓄量约束为

$$V_{\min_i} \leqslant V_{i,t} \leqslant V_{\max_i} \tag{6}$$

式中: $V_{i,t}$ 表示第 i 个水库第 t 时段的蓄量, 万 m^3; $V_{\min_i}$ 表示第 i 个水库的蓄量下限, 万 m^3; $V_{\max_i}$ 表示第 i 个水库的蓄量上限, 万 m^3。

设计分水量约束为

$$W_{FZ_m} \leqslant W_{FS_m} \tag{7}$$

式中: W_{FZ_m} 表示第 m 个分水口的总分水量, 万 m^3; W_{FS_m} 表示第 m 个分水口的设计分水量, 万 m^3。

需水量约束为

$$S_{j,t} \leqslant D_{j,t} \tag{8}$$

式中: $S_{j,t}$ 表示第 j 个用户第 t 时段接收的供水量, 万 m^3; $D_{j,t}$ 表示第 j 个用户第 t 时段的需水量, 万 m^3。

干渠输水能力约束为

$$C_{Z_m} \leqslant C_m \tag{9}$$

式中: C_{Z_m} 表示第 m 个分水口流向下一段干渠的水量, 万 m^3; C_m 表示第 m 个分水口所在干渠的设计输水水量, 万 m^3。

变量非负约束, 即所有变量非负限制。

2.3 模型求解

求解多目标优化配置模型, 一些学者采用智能优化算法进行求解, 如粒子群算法[11]、遗传算法[12]和蚁群算法[13]等。本文采用权重法将多目标优化模型转化为单目标模型求解, 并用 LINGO 软件求解单目标模型, 通过假定不同权重得到非劣解集。

在单目标优化过程中, 采用用户权重系数处理有限水资源量在不同用户之间的竞争性分配问题。用户权重系数反映有限水资源量在不同用户间分配时的优先级别与分配比例关系; 用户间的权重系数主要依据生活、生产、生态等不同用户优先供水的次序级别进行设定。

3 基于区间犹豫模糊语言与 TOPSIS 的多目标决策方法

本文采用基于区间犹豫模糊语言集与 TOPSIS 的多属性决策方法, 处理多目标非劣解集的优选问题。因为每个决策者对方案优劣的界定不完全一样, 所以该方法充分考虑到了主观性。此方法中区间数[14]的加入较之前的方法有很大的改进, 区间数可以为决策者的判断留有余地, 比单值更加灵活, 在扩大评价范围的同时也使得评价更具有代表性和说服力。区间数与犹豫模糊理论[15]的结合使该方法在主观因素上能更好地表达人们在决策时的犹豫心理, 降低了决策者主观判断出现失误的风险。在对水资源优化配置方案进行评价时, 决策专家往往需要考虑多个指标如缺水量、调水量等, 在对每个指标进行评价时用区间犹豫模糊语言来表达他们的犹豫心理可有效选择最优方案。

3.1 区间犹豫模糊语言概述

3.1.1 区间犹豫模糊语言集合

对于水资源多目标优化配置的非劣解集(方案集), 设 $X = \{x_i | i = 1, 2, \cdots, n\}$ 为待评价的方案集合, S 表示一个语言术语集合; $s_{\delta(x)}$ 表示对方案 x 的评价, 且 $s_{\delta(x)} \in S$。例如 $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$, 其中 s_3 为中立评价"一般", 其余分别为 s_0 表示"很差"、s_1 表示"差"、s_2 表示"较差"、s_4 表示"较好"、s_5 表示"好"、s_6 表示"很好"。则, 定义区间犹豫模糊语言集合[16-17]

$$A = \{\langle x, s_{\delta(x)}, H_A(x)\rangle | x \in X\} \tag{10}$$

其中 $H_A(x)$ 是有限个封闭区间数的集合且区间数范围为(0, 1], 表示 x 方案对应于 $s_{\delta(x)}$ 评价级别的可能性与配对程度。

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一个方案 x 的评价语言集合为 $\langle s_{\delta(x)}, H(x)\rangle$, 它是区间犹豫模糊语言集合中的一个元素, 称其为区间犹豫模糊语言数。如:

$$A=\{\langle x, s_{\delta(x)}, H_A(x)\rangle \mid x\in X\}=\left\{\begin{matrix}\langle x_1, s_2, \{[0.2, 0.3], [0.3, 0.4], [0.4, 0.5]\}\rangle, \\ \langle x_2, s_3, \{[0.4, 0.6], [0.7, 0.8], [0.8, 0.9]\}\rangle\end{matrix}\right\} \tag{11}$$

即为一个包含两个方案的区间犹豫模糊语言集合, $\langle x_2, s_3, \{[0.4, 0.6], [0.7, 0.8], [0.8, 0.9]\}\rangle$ 是其中一个区间犹豫模糊语言数, $[0.4, 0.6]$ 表示某个专家对 x_2 方案的某个指标评价为 s_3 的配对程度, 配对程度越接近 1 表示 x_2 对应于 s_3 的可能性越高, 配对程度越接近 0 表示 x_2 对应于 s_3 的可能性越低。其他区间数的含义与之相同。

3.1.2　评价术语的距离测度

设 S_α 和 S_β 是语言集合中的两个元素, 则定义 S_α 和 S_β 之间的距离测度[18] 为

$$d(S_\alpha, S_\beta)=\frac{|\alpha-\beta|}{2\mathrm{k}+1} \tag{12}$$

式中: k 表示语言术语集合 S 中的中立评价的下标; $2\mathrm{k}+1$ 表示语言术语集合 S 中评价级别的个数。

设 $b=[b^m, b^n]=\{x \mid b^m\leqslant x\leqslant b^n\}$, 则称 b 为一个区间数。对于区间数有如下的运算法则[19]:

区间数的加法

$$b_1+b_2=[b_1^m+b_2^m, b_1^n+b_2^n] \tag{13}$$

区间数的减法

$$b_1-b_2=[b_1^m-b_2^m, b_1^n-b_2^n] \tag{14}$$

3.2　区间犹豫模糊语言数距离测度

距离相似性测度[20-22] 是用距离刻画匹配实体之间相似性程度的一种定量度量指标。把距离相似性测度应用到区间犹豫模糊语言集合中, 能够度量两个方案之间的差距, 对比不同方案与理论最优解和最劣解的距离来寻求最优方案。基于以上理论, 本文运用距离相似性测度衡量同一个指标下两个方案的区间犹豫模糊语言数之间的距离。

3.2.1　距离相似性测度

设和是任意两个水资源优化配置方案, 对于区间犹豫模糊语言集合

$$A=\{\langle x_i, s_{\delta(x_i)}, H_A(x_i)\rangle x_1, x_2\} \tag{15}$$

其中 $H_A(x_i)=\{[\lambda_{it}^m, \lambda_{it}^n] \mid [\lambda_{it}^m, \lambda_{it}^n]\subset(0, 1], t=1, 2, \cdots, T\}$ 是按从大到小顺序排列的区间数组合。设每个方案都具有 c 个评价指标, 则 c 个评价指标的闵可夫斯基距离[23] 为

$$d(x_1, x_2)=\sqrt{\sum_{l=1}^{c}\left(\frac{|\delta(x_1)-\delta(x_2)|^2}{(2\mathrm{k}+1)^2}+\sum_{t=1}^{T}\left(|\lambda_{l1t}^m-\lambda_{l2t}^m|^2+|\lambda_{l1t}^n-\lambda_{l2t}^n|^2\right)\right)} \tag{16}$$

式中: λ_{lit}^m, λ_{lit}^n 为第 l 个指标下 x_i 评价方案的第 t 个区间数的下限和上限。使用上述距离测度公式的前提为区间犹豫模糊语言数中的区间数个数 T 相等。当区间数个数不相等时则向区间数数量少的犹豫模糊数中添加区间数至相等。

在运用 TOPSIS 法时, 需要根据决策矩阵找出最优方案(理论最优解)和最劣方案(理论最劣解), 这里采用一种给区间模糊语言数打分的方法从而在众多方案中找出最优方案与最劣方案。

3.2.2　语言尺度函数与打分函数

设 δ 是一个非负实数, 语言尺度函数 f 是从语言到实数的一个映射, 即 $f: s_i\rightarrow\delta_i\ (i=0, 1, 2, \cdots, 2\mathrm{k})$, $0\leqslant\delta_0<\delta_1<\cdots<\delta_{2\mathrm{k}}$。本文中使用的语言尺度函数[18] 为

$$f(s_i)=\delta_i=\frac{i}{2\mathrm{k}}\ (i=0, 1, 2, \cdots, 2\mathrm{k}), \delta_i\in[0, 1] \tag{17}$$

设 $\alpha=\langle s_{\delta(\alpha)}, H_\alpha\rangle=\langle s_{\delta(\alpha)}, \bigcup_{\lambda=[\lambda^m, \lambda^n]\in H_\alpha}\{[\lambda^m, \lambda^n]\}\rangle$ 是一个区间犹豫模糊语言数, 则 H_α 的期望为 $E_\lambda[H_\alpha]=\dfrac{\sum_{[\lambda^m, \lambda^n]\in H_\alpha}(\lambda^m, \lambda^n)}{2\cdot H_\alpha^\#}$, 其中 $H_\alpha^\#$ 是 H_α 中区间数的总个数。相应地, α 的打分函数[18] 为 $E(\alpha)=f(s_{\delta(\alpha)})\times E_\lambda(H_\alpha)$。打分函数值越大, 区间犹豫模糊语言数越大。

根据上述方法, 可计算每个指标各个方案对应的区间犹豫模糊语言数的打分函数, 通过打分函数的比较, 将每个指标下得分最高的方案的区间犹豫模糊语言数组合在一起, 得到理论最优解 x^+, 反之得到理论最劣解 x^-。

3.3　基于区间犹豫模糊语言集与 TOPSIS 的决策流程

TOPSIS 法[24-25] 又称为优劣解距离法, 其基本原理是通过度量评价方案与最优解、最劣解的距离来进行排序, 若评价方案最接近最优解同时又最远离最劣解则为最优。本文结合区间犹豫模糊语言数的距离相似性测度与 TOPSIS 法, 对水资源优化配置方案进行多指标评价决策。

在水资源优化配置方案遴选问题中, 设有 n 个非劣解方案 $X=\{x_1, x_2, \cdots, x_n\}$, 有 m 个指标 $Y=\{y_1, y_2, \cdots, y_m\}$, 有 z 个专家对各方案的每个指标进行评价。决策流程见图 4。

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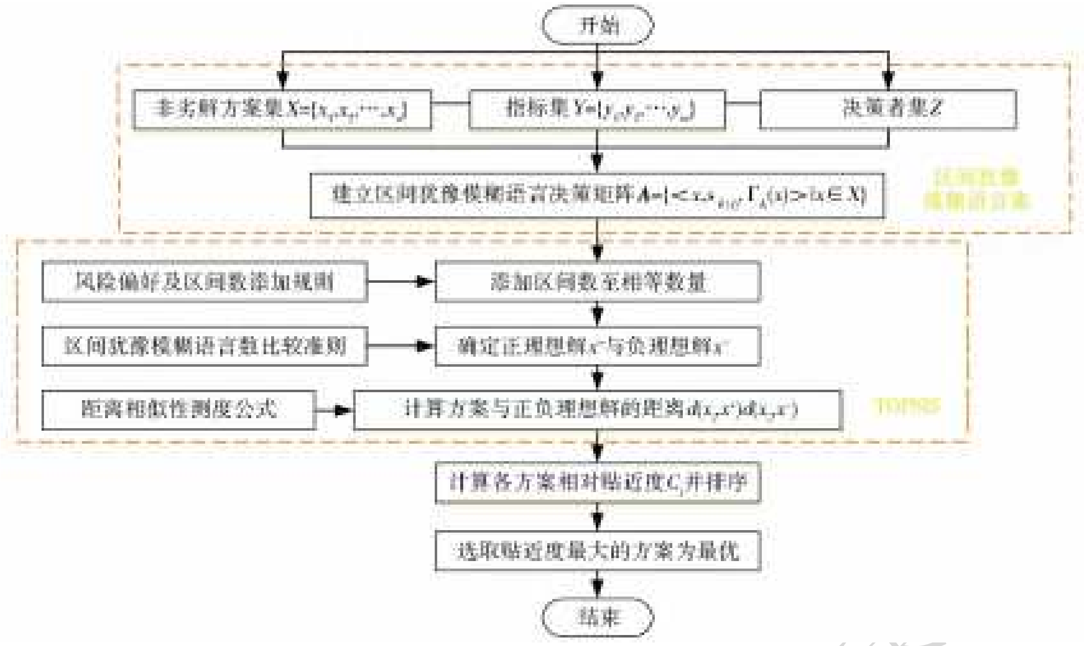

图 4　基于区间犹豫模糊语言集与 TOPSIS 的决策流程

Step 1 建立区间犹豫模糊语言决策矩阵。

Step 2 根据风险偏好及区间数添加规则，添加区间数使每个指标下各方案的区间数相等。

Step 3 根据区间犹豫模糊语言数的比较准则确定理论最优解与理论最劣解。

Step 4 根据距离相似性测度公式，计算每个方案与理论最优解和理论最劣解的距离。

Step 5 计算每个方案的相对贴近度 C_i，$C_i = d(x_i, x^-)/[d(x_i, x^+) + d(x_i, x^-)]$。对其进行排序，$C_i$ 越大，方案越优。

4　实例分析

4.1　非劣解生成

以 1956 年至 1961 年水文长系列资料计算，改变权重系数求解多目标模型得到非劣解集，最终得到 7 个非劣解方案，即 $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$。整理可得水资源优化配置方案集见表 1、表 2、图 5。

表 1　多目标权重配置方案

方案	年总缺水量	年总调水量
1	0.95	0.05
2	0.90	0.10
3	0.80	0.20
4	0.70	0.30
5	0.55	0.45
6	0.40	0.60
7	0.25	0.75

表 2　水资源优化配置方案结果

单位：亿 m^3

方案	年均缺水量	年均调水量
1	1.41	14.09
2	0.97	14.79
3	0.73	15.69
4	0.89	15.37
5	1.02	15.13
6	1.04	15.05
7	1.68	13.72

由图 5 可知，年均调水量与年均缺水量大致成反比关系：年均调水量越大，缺水量越小；年均调水量越小，缺水量就越大。因此，缺水量与调水量的目标相互冲突，需要综合考虑缺水量与调水量，对非劣解集进行筛别选取最优方案。

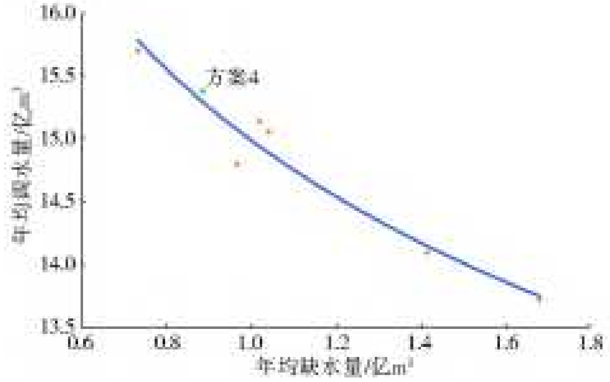

图 5　年均缺水量与调水量对比

4.2　水资源优化配置方案优选

从 7 个水资源优化配置方案中选取最优非劣解方案，从年均缺水量和年均调水量两个指标

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$Y=\{y_1, y_2\}$对这7个方案进行评价。语言集$S=\{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$中的元素分别表示非常差、差、较差、一般、较好、好、非常好。决策专家根据主观经验和客观因素对7个方案的每个指标进行评价，评价结果以区间犹豫模糊语言决策矩阵的形式给出。

经过讨论与协商，最终的区间犹豫模糊语言决策矩阵见表3。

表3　区间犹豫模糊语言决策矩阵

方案	y_1	y_2
x_1	$\langle s_2, \{[0.6, 0.9]\}\rangle$	$\langle s_4, \{[0.4, 0.7], [0.7, 0.9]\}\rangle$
x_2	$\langle s_4, \{[0.3, 0.5], [0.7, 0.9]\}\rangle$	$\langle s_3, \{[0.4, 0.6], [0.6, 0.7]\}\rangle$
x_3	$\langle s_5, \{[0.4, 0.6], [0.6, 0.8]\}\rangle$	$\langle s_1, \{[0.3, 0.7]\}\rangle$
x_4	$\langle s_4, \{[0.8, 0.9]\}\rangle$	$\langle s_2, \{[0.5, 0.8], [0.8, 0.9]\}\rangle$
x_5	$\langle s_3, \{[0.2, 0.4], [0.4, 0.5], [0.6, 0.8]\}\rangle$	$\langle s_4, \{[0.7, 0.8]\}\rangle$
x_6	$\langle s_3, \{[0.2, 0.3], [0.3, 0.4], [0.6, 0.7]\}\rangle$	$\langle s_3, \{[0.1, 0.3], [0.3, 0.5], [0.6, 0.8]\}\rangle$
x_7	$\langle s_1, \{[0.3, 0.6], [0.7, 0.9]\}\rangle$	$\langle s_5, \{[0.1, 0.3], [0.4, 0.7]\}\rangle$

根据水资源优化配置默认风险规避原则，添加最小的区间数，使各个区间犹豫模糊语言数的区间数个数相等，处理结果见表4。

表4　正规化区间犹豫模糊语言决策矩阵

方案	y_1	y_2
x_1	$\langle s_2, \{[0.6, 0.9], [0.6, 0.9], [0.6, 0.9]\}\rangle$	$\langle s_4, \{[0.4, 0.7], [0.7, 0.9], [0.4, 0.7]\}\rangle$
x_2	$\langle s_4, \{[0.3, 0.5], [0.7, 0.9], [0.3, 0.5]\}\rangle$	$\langle s_3, \{[0.4, 0.6], [0.6, 0.7], [0.4, 0.6]\}\rangle$
x_1	$\langle s_5, \{[0.4, 0.6], [0.6, 0.8], [0.4, 0.6]\}\rangle$	$\langle s_1, \{[0.3, 0.7], [0.3, 0.7], [0.3, 0.7]\}\rangle$
x_4	$\langle s_4, \{[0.8, 0.9], [0.8, 0.9], [0.8, 0.9]\}\rangle$	$\langle s_2, \{[0.5, 0.8], [0.8, 0.9], [0.5, 0.8]\}\rangle$
x_5	$\langle s_3, \{[0.2, 0.4], [0.4, 0.5], [0.6, 0.8]\}\rangle$	$\langle s_4, \{[0.7, 0.8], [0.7, 0.8], [0.7, 0.8]\}\rangle$
x_6	$\langle s_3, \{[0.2, 0.3], [0.3, 0.4], [0.6, 0.7]\}\rangle$	$\langle s_3, \{[0.1, 0.3], [0.3, 0.5], [0.6, 0.8]\}\rangle$
x_7	$\langle s_1, \{[0.3, 0.6], [0.7, 0.9], [0.3, 0.6]\}\rangle$	$\langle s_5, \{[0.1, 0.3], [0.4, 0.7], [0.1, 0.3]\}\rangle$

计算每个方案下缺水量与调水量区间犹豫模糊语言数的打分函数，见表5。

表5　区间犹豫模糊语言数打分函数

方案	y_1	y_2
x_1	0.250	0.422
x_2	0.356	0.275
x_3	0.472	0.083
x_4	0.567	0.239
x_5	0.242	0.500
x_6	0.208	0.217
x_7	0.094	0.264

通过表5可以确定理论最优解和理论最劣解。

$x^+ = \langle s_4, \{[0.8, 0.9], [0.8, 0.9], [0.8, 0.9]\}\rangle$, $\langle s_4, \{[0.7, 0.8], [0.7, 0.8], [0.7, 0.8]\}\rangle$;

$x^- = \langle s_1, \{[0.3, 0.6], [0.7, 0.9], [0.3, 0.6]\}\rangle$, $\langle s_1, \{[0.3, 0.7], [0.3, 0.7], [0.3, 0.7]\}\rangle$。

计算各个方案与理论最优解和理论最劣解的距离及其相对贴近度，计算结果见表6。

相对贴近度C_i越小，表明该方案与理论最优解的相对距离越近，与理论最劣解的相对距离越远，由此可得方案优劣排序如下：

$x_4 > x_1 > x_5 > x_2 > x_7 > x_6 > x_3$，所以最优方案是$x_4$。方案$x_4$的年均缺水量较小，年均调水量也处在一个相对合理的水平上。

表6　各方案与理论最优最劣解的距离及其相对贴近度

距离及其相对贴近度	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$d(x_i, x^+)$	0.641 6	1.063 2	1.124 3	0.426 2	1.000 2	1.486 9	1.488 0
$d(x_i, x^-)$	0.783 6	0.644 4	0.605 4	1.133 2	0.930 2	0.832 6	0.858 2
C_i	0.450 2	0.622 6	0.650 0	0.273 3	0.518 1	0.641 0	0.634 2

5　结　论

工程调水条件下的水资源多目标优化配置需要统筹协调本地水与工程调水的关系，在优选非劣解时对两个向相反方向优化的指标进行评价，定义区间犹豫模糊语言数的距离测度，将之与传统的

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TOPSIS 方法相结合,用基于区间犹豫模糊语言集与 TOPSIS 的多指标决策方法优选非劣解集,主要结论如下:(1)经过对鄂北水资源配置方案的遴选,选择 4 号方案为最优水量调度方案。4 号方案侧重于降低缺水量的同时兼顾调水量,具有较高的合理性和可行性,对于鄂北水资源配置工程的实际应用有较高的参考价值。(2)基于区间犹豫模糊语言集与TOPSIS 的水资源多目标决策方法经本文实例验证具有较好的稳定性。

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Multi-objective decision-making of water resources based on interval hesitant fuzzy language set

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Abstract: Taking the water resources allocation project in the northern Hubei as the object, the multi-water source multi-objective optimal allocation was studied under the background of a large-scale water transfer project. The reservoirs in the northern Hubei are divided into three types: filling reservoirs, compensation reservoirs, and online reservoirs. The water resources allocation project in the northern Hubei is systematically generalized according to the characteristics of various reservoirs. A multi-water-source multi-objective optimal allocation model is established with the minimum annual water shortage and the minimum water transfer cost as the objective function. The weight coefficient method is used to obtain the Pareto optimal set. The multi-objective decision-making method based on interval hesitant fuzzy language and TOPSIS is adopted to evaluate the water shortage and water transfer of each scheme and select the optimal scheme. The results show that: for the water resources allocation project in the northern Hubei, the scheme with a weight ratio of water shortage to water transfer being 7∶3 is the best in comprehensive evaluation, which focuses on water shortage while the water transfer cost is taken into account; the multi-objective decision-making method of water resources based on interval hesitant fuzzy language set and TOPSIS has been proved to have high stability and feasibility by examples.

Key words: water transfer project; optimal allocation; multi-objective; hesitant fuzzy language; TOPSIS

The optimal allocation of water resources is an important measure for the rational and efficient use of water resources, which is also a key issue in the research of water resources[1-3]. The researches[4-5] on the optimal allocation of water resources in China and abroad has made great progress, from single-objective to multi-objective, namely that it shifts from a single pursuit of economic goals to comprehensive consideration of social, economic, environmental, and other goals, and from an optimal dispatch of a single water source to a complex configuration of joint optimal dispatch of multiple water sources. Zhang et al.[6] established the multi-objective optimal allocation model in the main stream section of the Yellow river, and used an improved genetic algorithm to solve the optimal allocation of the river section. Yan et al.[7] established a multi-objective optimal model with the goal of

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minimizing the weighted relative total water shortage depth and the total cost of system water supply, which was solved by multi-objective particle swarm optimization (MOPSO). Gu et al.[8] established a multi-objective risk analysis model and proposed a multi-objective risk assessment method for optimal allocation of water resources based on stochastic simulation technology. Based on the establishment of a coupling quantitative model for the joint distribution of water supply and demand in the irrigated area, Fu et al.[9] analyzed the encounter probability of water shortage and then constructed an interval linear programming model. According to the demand and total control of regional water and the pollution control requirements in the water function zone, Gao et al.[10] established an optimal allocation model of regional water resources based on the dual control of water quantity and quality, calculated the allocation benefits with AHP method, entropy weight method, and AHP-entropy weight coupled method under different scenarios and then selected the optimal allocation scheme. But from the existing literature, the multi-objective decision-making method is still under development. This paper intends to build a multi-water-source multi-objective optimal allocation model with the minimum annual water shortage and the minimum water transfer cost as the objective function under the background of a large-scale water transfer project. The weight coefficient method is used to solve the optimal allocation model and obtain the Pareto optimal set. The multi-objective decision-making method based on interval hesitant fuzzy language and TOPSIS is adopted to select the optimal scheme. Taking allocation project of water resources in the northern Hubei as the object, we carry out case studies and recommend the optimal scheme of water transfer.

1 Generalization of water resources allocation project

The water resources allocation project in the northern Hubei uses the Qingquangou tunnel to divert water from the Danjiangkou reservoir. Without affecting the scale and process of the water transfer in the middle route of the South-to-North Water Diversion Project, this project has solved the problem of water shortages in the northern Hubei and meets the water demand for living, production and ecology in the water-receiving area of the northern Hubei.

The optimal allocation model of water resources in the northern Hubei is relatively complex, including multiple water sources, multiple users, and multiple nodes. The element generalization is mainly divided into three forms: nodes, reservoirs, and users. Nodes are divided into storage nodes, subsiding water nodes, main canal control nodes, and water diversion nodes at the bleeder. Reservoirs are divided into filling reservoirs, compensation reservoirs (virtual reservoirs set up when the bleeder of general main canal is of direct supply and compensation), and online reservoirs. Users are divided into domestic water, ecological water, industrial water, and agricultural water.

1.1 Filling reservoir

Filling reservoir means that the reservoir will receive the water diverted from the bleeder. This part of water will directly enter the filling reservoir, and will be adjusted and distributed with the natural water, and then supplied to the users. The specific generalization is shown in Fig. 1.

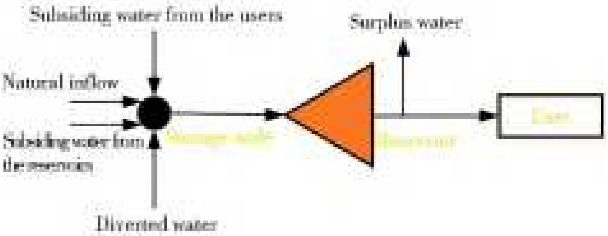

Fig. 1 Generalization of filling reservoir model

The balance equation of water quantity of the reservoir is

$$V_{i,t+1} = V_{i,t} + W_{R_{i,t}} + W_{K_{i,t}} + W_{Y_{i,t}} + W_{T_{i,t}} - O_{i,t} - S_{S_{i,t}} \qquad (1)$$

where: $V_{i,t+1}$ and $V_{i,t}$ represent the storage capacity of the i-th reservoir at moments $t+1$ and t, respectively; $W_{R_{i,t}}$ represents the natural inflow of the i-th reservoir during the t-th period; $W_{K_{i,t}}$ represents subsiding water of the i-th reservoir received from other reservoirs during the t-th period; $W_{Y_{i,t}}$ represents the diverted water received by the i-th reser-

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voir during the t-th period; $W_{T_{i,t}}$ represents the subsiding water of users received by the i-th reservoir during the t-th period; $Q_{i,t}$ represents the surplus water received by the i-th reservoir during the t-th period; $S_{S_{i,t}}$ represents the water supply of the i-th reservoir during the t-th period.

In addition, for the restriction of the storage capacity of the filling reservoir, the diverted water volume should be added to the storage capacity to further restrict the storage capacity, and the following storage capacity constraint should be met:

$$V_{\max_i} \quad V_{i,t+1} + W_{YK_{i,t}} \tag{2}$$

where: $W_{YK_{i,t}}$ represents the diverted water volume of the i-th reservoir during the t-th period.

1.2 Compensation reservoir

Compensation reservoir means that the reservoir will receive the water diverted from the bleeder, but this part of water cannot automatically flow into the reservoir for regulation, but be diverted to the downstream channel of the compensation reservoir, which is supplied to downstream users together with the local runoff adjusted by the reservoir. Therefore, the compensation reservoir mainly implements compensation for water diversion by readjusting the local runoff. In the modeling process, a virtual reservoir with a storage capacity of 0 (namely, diversion channel) is established in the downstream of compensation reservoir, and the diverted water at the bleeder does not enter the compensation reservoir but is directly supplied to the corresponding users through the virtual reservoir, namely that the water is supplied to the users in this area by the compensation reservoir and the corresponding virtual reservoir. The specific model generalization is shown in Fig. 2.

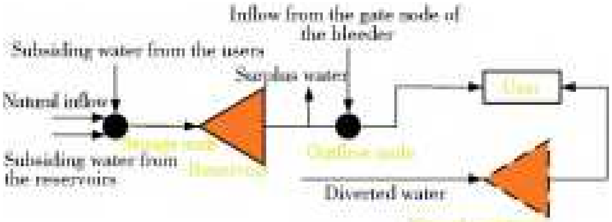

Fig. 2   Generalization of compensation reservoir model

1.3 Online reservoir (Fengjiangkou reservoir)

Online reservoir is above the main canal channel. All main canal water and its own natural inflow will directly enter the reservoir for comprehensive regulation and storage, and then be supplied to the corresponding users by the online reservoir. Other outflow water (including surplus water) will continue to flow along the main canal to the downstream river. Therefore, as a transfer station, the online reservoir is very important for the impoundment of main canal water. The specific generalization of online reservoir model is shown in Fig. 3.

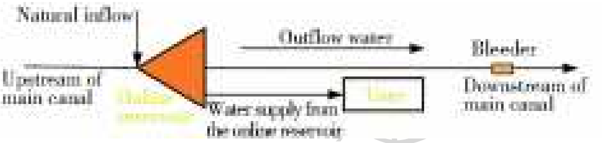

Fig. 3   Generalization of online reservoir model

The water resources allocation project in this area is generalized into two parts, namely, elements and relationships. After generalization, the final elements include 79 nodes, 196 reservoirs, 11 virtual reservoirs, and 148 users.

2 Multi-water-source multi-objective optimal allocation model

The northern Hubei water resources allocation project uses the Qingquangou tunnel to divert water from Danjiangkou reservoir into two lines, namely, the Yindan general main canal and the main diversion canal of northern Hubei. The Yindan general main canal supplies water to the reservoirs in the Tangxi region, and then supplies water to users in each area through each reservoir. The main diversion canal in the northern Hubei supplies water to the reservoirs along the northern Hubei area through the bleeders and then supplies water to users in each area through each reservoir. The water resources allocation system of northern Hubei is a large and complex system, which requires the coordination of three aspects. The first aspect is the relationship between local water and diverted water, which focuses on the water transfer cost, the determination of water transfer time, and the distribution of water transfer in each water-receiving area. The second aspect is the mutual compensation relationship between various reservoirs, which mainly determines the optimal water supply

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sequence of each reservoir so as to improve the utilization rate of local water resources. The third aspect is the relationship between water supply and water demand of the reservoirs, which uses the regulation and storage of the reservoirs so that the water supply process can meet the requirements of the water demand process as much as possible. For such a complex, multi-water-source, multi-objective, and multi-user allocation problem, the first task is to build an overall large-scale and multi-objective optimization model. Through the application optimization of water transfer projects in the region, priority should be given to local water resources and water transfer reduction, so as to achieve the dual goals of the minimum water shortage and the minimum water transfer cost.

2.1 Objective function

Considering the social and economic goals of the optimal allocation of water resources, the minimum total annual water shortage and the minimum water transfer cost are adopted to reflect social goal and economic goal respectively.

Aim to minimize the annual total water shortage

$$F_1=\min\sum_{j=1}^{J}\sum_{t=1}^{T}a_j\cdot(D_{j,t}-S_{j,t})\qquad(3)$$

where: a_j is the weight coefficient of the j-th user, indicating the importance of each user; $D_{j,t}$ represents the water demand (10^4 m^3) of the j-th user during the t-th period; $S_{j,t}$ represents the amount of water (10^4 m^3) received by the j-th user during the t-th period.

Aim to minimize the water diversion project cost

$$F_2=\min\beta\sum_{i=1}^{I}\sum_{t=1}^{T}Y_{i,t}\qquad(4)$$

where: $Y_{i,t}$ represents the water transfer quantity (10^4 m^3) of the i-th reservoir during the t-th period; β is the water transfer cost (CNY $10^4/10^4$ m^3).

Using the weight method, we can convert the multi-objective optimization problem into a single-objective optimization problem:

$$F=\min[\lambda F_1+(1-\lambda)F_2]\qquad(5)$$

where: λ is the weight coefficient of water shortage and indicates the importance of water shortage relative to water transfer.

2.2 Constraints

Constraint of reservoir storage capacity

$$V_{\min_i}\leqslant V_{i,t}\leqslant V_{\max_i}\qquad(6)$$

where: $V_{i,t}$ represents the storage capacity of the i-th reservoir during the t-th period (10^4 m^3); $V_{\min_i}$ represents the lower limit of the storage capacity of the i-th reservoir (10^4 m^3); $V_{\max_i}$ represents the upper limit of the storage capacity of the i-th reservoir (10^4 m^3).

Constraint of design water diversion

$$W_{FZ_m}\leqslant W_{FS_m}\qquad(7)$$

where: W_{FZ_m} represents the total water diversion of the m-th bleeder (10^4 m^3); W_{FS_m} represents the design water diversion of the m-th bleeder (10^4 m^3).

Water demand constraint

$$S_{j,t}\leqslant D_{j,t}\qquad(8)$$

where: $S_{j,t}$ represents the water supply received by the j-th user during the t-th period (10^4 m^3); $D_{j,t}$ represents the water demand of the j-th user during the t-th period (10^4 m^3).

Constraints on the water delivery capacity of main canal

$$Cz_m\leqslant C_m\qquad(9)$$

where: Cz_m represents the amount of water flowing from the m-th bleeder to the next section of the main canal (10^4 m^3); C_m represents the designed water delivery volume of the main canal where the m-th bleeder is located (10^4 m^3).

Non-negative variable constraints, i.e All variables are non-negative.

2.3 Model solution

To solve the multi-objective optimal allocation model, some scholars choose intelligent optimization algorithms, such as particle swarm optimization[11], genetic algorithm[12], and ant colony algorithm[13]. This paper uses the weight method to transform the multi-objective optimization model into the single-objective model, and adopts the LINGO software to solve the single-objective model. Finally, the pareto optimal set is obtained by assuming different weights.

In the single-objective optimization process, the user weight coefficient is used to deal with the competitive allocation of limited water resources among different users, which reflects the priority and distribution proportion of the limited water resources among different users. The weight coeffi-

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cients are mainly set according to the priority order of water supply for different users, such as life, production, and ecology.

3 Multi-objective decision-making method based on interval hesitant fuzzy language and TOPSIS

This paper adopts a multi-attribute decision-making method based on interval hesitant fuzzy language sets and TOPSIS to deal with the optimization problem of multi-objective Pareto optimal set. Because each decision maker has different definitions for the quality of schemes, the above method fully takes into account the subjectivity. The interval number[14] introduced in this method is a great improvement over the previous methods, which can leave room for the judgment of decision maker and is more flexible than a single value. In addition, it expands the evaluation scope and makes the evaluation more representative and convincing. The combination of interval numbers and hesitant fuzzy theory[15] enables this method to better express the hesitation of people in decision-making from a perspective of subjectivity, and reduces the failure risk of subjective judgments. When evaluating the optimal allocation schemes of water resources, decision-making experts often need to consider multiple indices such as water shortage and water transfer. When each index is evaluated, using interval hesitating fuzzy language to express their hesitation can effectively choose the optimal scheme.

3.1 Overview of interval hesitant fuzzy language

3.1.1 Interval hesitant fuzzy language set

For the Pareto optimal set (scheme set) of the multi-objective optimal allocation of water resources, $X = \{x_i | i = 1, 2, \cdots, n\}$ is set as the set of schemes to be evaluated; S represents a set of language terms; $s_{\theta(x)}$ represents the evaluation of the scheme x, and $s_{\theta(x)} \in S$. For example, $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$, where: s_3 refers to the neutral evaluation "general", and s_0, s_1, s_2, s_4, s_5 and s_6 refer to "very poor", "poor", "worse", "good", "better" and "very good" respectively. Thus, the interval hesitant fuzzy language set[16-17] is defined as follows:

$$A = \{\langle x, s_{\theta(x)}, \mathrm{H}_A(x)\rangle | x \in X\} \tag{10}$$

where: $H_A(x)$ is a set of a finite number of closed intervals and the range of interval numbers is $(0, 1]$, indicating the possibility and degree of the scheme x matching evaluation level $s_{\theta(x)}$.

The evaluation language set of scheme x is $\langle s_{\delta(x)}, H(x)\rangle$, which is an element in the set of interval hesitant fuzzy languages and is called the number of interval hesitant fuzzy languages. For example,

$$A = \{\langle x, s_{\theta(x)}, H_A(x)\rangle | x \in X\} = \left\{\begin{array}{l}\langle x_1, s_2, \{[0.2, 0.3], [0.3, 0.4], [0.4, 0.5]\}\rangle, \\ \langle x_2, s_3, \{[0.4, 0.6], [0.7, 0.8], [0.8, 0.9]\}\rangle\end{array}\right\} \tag{11}$$

It is an interval hesitant fuzzy language set containing two schemes, in which $\langle x_2, s_3, \{[0.4, 0.6], [0.7, 0.8], [0.8, 0.9]\}\rangle$ is the number of interval hesitant fuzzy languages in one scheme and $[0.4, 0.6]$ indicates the pairing degree that an expert evaluates a certain index of the scheme x_2 as s_3. The pairing degree closer to 1 means that x_2 is more likely to correspond to s_3, and the pairing degree closer to 0 means that x_2 is less likely to correspond to s_3. The meaning of other interval numbers is the same.

3.1.2 Distance measure of evaluation terms

Assuming that S_α and S_β are two elements in the language set, the distance measure[18] between S_α and S_β is defined as

$$d(S_\alpha, S_\beta) = \frac{|\alpha - \beta|}{2k + 1} \tag{12}$$

where: k represents the subscript of the neutral evaluation in the language term set S, and $2k + 1$ represents the number of evaluation levels in the language term set S.

It is assumed that $b = [b^m, b^n] = \{x | b^m \leqslant x \leqslant b^n\}$, where: b is called an interval number. For interval numbers, there are the following algorithms[19]:

Addition of interval numbers

$$b_1 + b_2 = [b_1^m + b_2^m, b_1^n + b_2^n] \tag{13}$$

Subtraction of interval numbers

$$b_1 - b_2 = [b_1^m - b_2^m, b_1^n - b_2^n] \tag{14}$$

3.2 Distance measure of interval hesitant fuzzy language number

The distance similarity measure[20-22] is a quan-

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titative measure index that uses distance to describe the degree of similarity between matching entities, which can be applied to the interval hesitant fuzzy language set to measure the difference between the two schemes. The distance between different solutions and the theoretical optimal/worst solution is compared to find the optimal scheme. Based on the above theory, this paper uses the distance similarity measure to measure the distance of interval hesitant fuzzy language numbers in two schemes under the same index.

3.2.1 Distance similarity measure

Assuming that x_1 and x_2 are two arbitrary optimal allocation schemes of water resources, for the interval hesitant fuzzy language

$$A=\{\langle x_i, s_{\delta(x_i)}, H_A(x_i)\rangle x_1, x_2\} \tag{15}$$

$H_A(x_i)=\{[\lambda_{lt}^m, \lambda_{lt}^n] \mid [\lambda_{lt}^m, \lambda_{lt}^n]\subset(0,1], t=1,2,\cdots,T\}$ is the combination of interval numbers arranged in descending order. Assuming that each scheme has c evaluation indices, the Minkowski distance[23] of c evaluation indices is

$$d(x_1, x_2)=\sqrt{\sum_{l=1}^{c}\left(\frac{|\delta(x_1)-\delta(x_2)|^2}{(2k+1)^2}+\sum_{t=1}^{T}(|\lambda_{l1t}^m-\lambda_{l2t}^m|^2+|\lambda_{l1t}^n-\lambda_{l2t}^n|^2)\right)} \tag{16}$$

where: λ_{lt}^m and λ_{lt}^n represent the lower limit and upper limit of the t-th interval number in the evaluation scheme x_i under the l-th index. The premise of using the above distance measure formula is that the number of intervals T in the interval hesitant fuzzy language number is equal. When the number of intervals is not equal, the hesitant fuzzy number with a small number of intervals should be added until it has the same interval number as others.

When using the TOPSIS method, we should find the optimal scheme (theoretical optimal solution) and the worst scheme (theoretical worst solution) according to the decision matrix. In this paper, a scoring method of the interval fuzzy language number is used to find the best scheme and the worst scheme among many schemes.

3.2.2 Language scaling function and scoring function

Assuming that δ_i is set as a non-negative real number, the language scaling function f is a mapping from language to real numbers, namely $f: s_i \rightarrow \delta_i (i=0, 1, 2, \cdots, 2k)$, $0\leqslant\delta_0<\delta_1<\cdots<\delta_{2k}$. The language scaling function[18] used in this paper is as follows:

$$f(s_i)=\delta_i=\frac{i}{2k}(i=0,1,2,\cdots,2k), \delta_i\in[0,1] \tag{17}$$

Assuming that $\alpha=\langle s_{\delta(\alpha)}, H_\alpha\rangle=\langle s_{\delta(\alpha)}, \bigcup_{\lambda=[\lambda^m,\lambda^n]\in H_\alpha}\{[\lambda^m, \lambda^n]\}\rangle$ is an interval hesitant fuzzy language number, the expectation of H_α is $E_\lambda[H_\alpha]=\frac{\sum_{[\lambda^m,\lambda^n]\in H_\alpha}(\lambda^m, \lambda^n)}{2\cdot H_\alpha^\#}$, where $H_\alpha^\#$ is the total number of interval numbers in H_α. Correspondingly, the scoring function of α[18] is $E(\alpha)=f(s_{\delta(\alpha)})\times E_\lambda(H_\alpha)$. A larger scoring function value leads to the greater interval hesitant fuzzy language number.

According to the above method, the scoring function of the interval hesitant fuzzy language number of each index corresponding to each scheme can be calculated. Through the comparison of the scoring functions, the interval hesitant fuzzy language number of the scheme with the highest score under each index is combined to obtain the theoretical optimal solution x^+. Otherwise, the theoretical worst solution x^- can be obtained.

3.3 Decision-making process based on interval hesitant fuzzy language set and TOPSIS

The TOPSIS method[24-25] is also called the Technique for Order Preference by Similarity to an Ideal Solution, whose basic principle is to sort the evaluation schemes by measuring the distance between them and the optimal/worst solution. If an evaluation scheme is the closest to the optimal solution and the farthest from the worst solution, it is the best. This paper combines the distance similarity measure of the interval hesitant fuzzy language number and the TOPSIS method to carry out multi-index evaluation and decision making of the optimal allocation schemes of water resources.

In the selection problem of the optimal allocation schemes of water resources, there are n Pareto optimal solutions $X=\{x_1, x_2, \cdots, x_n\}$, m indices $Y=\{y_1, y_2, \cdots, y_m\}$, and z experts evaluating each index of each scheme. The decision-making process is shown in Fig. 4.

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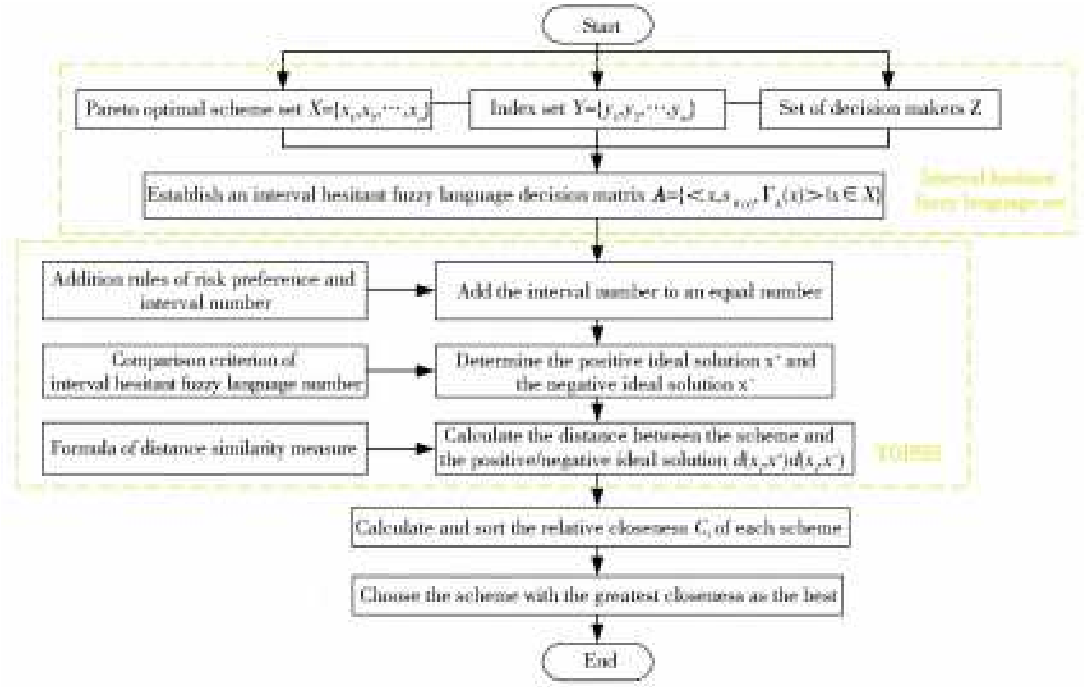

Fig. 4 Decision-making process based on interval hesitant fuzzy language set and TOPSIS

Step 1 An interval hesitant fuzzy language decision matrix is established.

Step 2 According to the addition rules of risk preference and interval number, interval number of each scheme is added to make sure the schemes have the same interval number under each index.

Step 3 The theoretical optimal/worst solutions are determined according to the comparison criterion of the interval hesitant fuzzy language number.

Step 4 According to the formula of distance similarity measure, the distance between each scheme and the theoretical optimal/worst solution is calculated.

Step 5 The relative closeness C_i of each scheme is calculated and sorted, with C_i, $C_i=\dfrac{d(x_i,x^-)}{d(x_i,x^+)+d(x_i,x^-)}$. A larger C_i means a better scheme.

4 Case analysis

4.1 Generation of Pareto optimal solution

Based on the long series of hydrological data from 1956 to 1961, the weight coefficients are changed to solve the multi-objective model and obtain the Pareto optimal set, and seven Pareto optimal solutions are obtained finally, namely, $X=\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$. The optimal allocation scheme sets of water resources after arrangement are shown in Tab. 1, Tab. 2, and Fig. 5.

Tab. 1 Multi-objective weight allocation schemes

Scheme	Total annual water shortage	Total annual water transfer
1	0.95	0.05
2	0.90	0.10
3	0.80	0.20
4	0.70	0.30
5	0.55	0.45
6	0.40	0.60
7	0.25	0.75

Tab. 2 Results of optimal allocation schemes of water resources

Unit: 100 million m³

Scheme	Average annual water shortage	Average annual water transfer
1	1.41	14.09
2	0.97	14.79
3	0.73	15.69
4	0.89	15.37
5	1.02	15.13
6	1.04	15.05
7	1.68	13.72

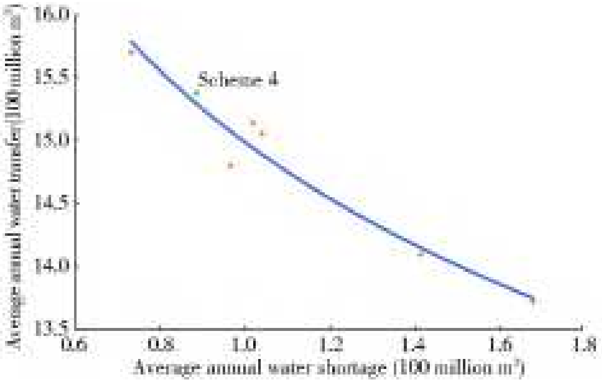

Fig. 5 Comparison of average annual water shortage and water transfer

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It can be seen from Fig. 5 that the average annual water transfer is roughly inversely proportional to the average annual water shortage. The larger average annual water transfer means the smaller water shortage. The smaller average annual water transfer means the greater water shortage. The goals of water shortage and water transfer conflict with each other, and it is necessary to screen the Pareto optimal set to select the optimal solution through comprehensive consideration of water shortage and water transfer.

4.2 Selection of optimal allocation schemes of water resources

This section selects the Pareto optimal scheme from seven optimal allocation schemes of water resources. These seven schemes are evaluated by the two indices $Y = (y_1, y_2)$, namely the average annual water shortage and the average annual water transfer. The elements in language set $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ represent very poor, poor, worse, general, good, better, and very good respectively. Based on subjective experience and objective factors, the decision-making experts evaluate each index of these seven schemes and give the evaluation results in the form of an interval hesitant fuzzy language decision matrix.

After discussion and negotiation, the final interval hesitant fuzzy language decision matrix is shown in Tab. 3.

Tab. 3 Interval hesitant fuzzy language decision matrix

Scheme	y_1	y_2
x_1	$\langle s_2, \{[0.6, 0.9]\}\rangle$	$\langle s_4, \{[0.4, 0.7], [0.7, 0.9]\}\rangle$
x_2	$\langle s_4, \{[0.3, 0.5], [0.7, 0.9]\}\rangle$	$\langle s_3, \{[0.4, 0.6], [0.6, 0.7]\}\rangle$
x_3	$\langle s_5, \{[0.4, 0.6], [0.6, 0.8]\}\rangle$	$\langle s_1, \{[0.3, 0.7]\}\rangle$
x_4	$\langle s_4, \{[0.8, 0.9]\}\rangle$	$\langle s_2, \{[0.5, 0.8], [0.8, 0.9]\}\rangle$
x_5	$\langle s_3, \{[0.2, 0.4], [0.4, 0.5], [0.6, 0.8]\}\rangle$	$\langle s_4, \{[0.7, 0.8]\}\rangle$
x_6	$\langle s_3, \{[0.2, 0.3], [0.3, 0.4], [0.6, 0.7]\}\rangle$	$\langle s_3, \{[0.1, 0.3], [0.3, 0.5], [0.6, 0.8]\}\rangle$
x_7	$\langle s_1, \{[0.3, 0.6], [0.7, 0.9]\}\rangle$	$\langle s_5, \{[0.1, 0.3], [0.4, 0.7]\}\rangle$

According to the default risk avoidance principle of optimal allocation of water resources, the minimum interval number is added to make sure that the interval number of hesitant fuzzy languages in each interval is equal. The processing results are shown in Tab. 4.

Tab. 4 Normalized interval hesitant fuzzy language decision matrix

Scheme	y_1	y_2
x_1	$\langle s_2, \{[0.6, 0.9], [0.6, 0.9], [0.6, 0.9]\}\rangle$	$\langle s_4, \{[0.4, 0.7], [0.7, 0.9], [0.4, 0.7]\}\rangle$
x_2	$\langle s_4, \{[0.3, 0.5], [0.7, 0.9], [0.3, 0.5]\}\rangle$	$\langle s_3, \{[0.4, 0.6], [0.6, 0.7], [0.4, 0.6]\}\rangle$
x_1	$\langle s_5, \{[0.4, 0.6], [0.6, 0.8], [0.4, 0.6]\}\rangle$	$\langle s_1, \{[0.3, 0.7], [0.3, 0.7], [0.3, 0.7]\}\rangle$
x_4	$\langle s_4, \{[0.8, 0.9], [0.8, 0.9], [0.8, 0.9]\}\rangle$	$\langle s_2, \{[0.5, 0.8], [0.8, 0.9], [0.5, 0.8]\}\rangle$
x_5	$\langle s_3, \{[0.2, 0.4], [0.4, 0.5], [0.6, 0.8]\}\rangle$	$\langle s_4, \{[0.7, 0.8], [0.7, 0.8], [0.7, 0.8]\}\rangle$
x_6	$\langle s_3, \{[0.2, 0.3], [0.3, 0.4], [0.6, 0.7]\}\rangle$	$\langle s_3, \{[0.1, 0.3], [0.3, 0.5], [0.6, 0.8]\}\rangle$
x_7	$\langle s_1, \{[0.3, 0.6], [0.7, 0.9], [0.3, 0.6]\}\rangle$	$\langle s_5, \{[0.1, 0.3], [0.4, 0.7], [0.1, 0.3]\}\rangle$

The scoring function of the interval hesitant fuzzy language number of water shortage and water transfer under each scheme is calculated and shown in Tab. 5.

According to Tab. 5, the theoretical optimal scheme x^+ and the theoretical worst scheme x^- can be determined.

$x^+ = \langle s_4, \{[0.8, 0.9], [0.8, 0.9], [0.8, 0.9]\}\rangle, \langle s_4, \{[0.7, 0.8], [0.7, 0.8], [0.7, 0.8]\}\rangle;$

$x^- = \langle s_1, \{[0.3, 0.6], [0.7, 0.9], [0.3, 0.6]\}\rangle, \langle s_1, \{[0.3, 0.7], [0.3, 0.7], [0.3, 0.7]\}\rangle$。

The distance and relative closeness between each scheme and the theoretical optimal/worst solution are calculated. The calculation results are

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shown in Tab. 6.

Tab. 5 Scoring function of interval hesitant fuzzy language number

Scheme	y_1	y_2
x_1	0.250	0.422
x_2	0.356	0.275
x_3	0.472	0.083
x_4	0.567	0.239
x_5	0.242	0.500
x_6	0.208	0.217
x_7	0.094	0.264

A smaller relative closeness C_i means that the relative distance between the scheme and the theoretical optimal solution is smaller, and the relative distance between the scheme and the theoretical worst solution is larger. Therefore, the quality of the schemes is ranked as follows: $x_4 > x_1 > x_5 > x_2 > x_7 > x_6 > x_3$, and the optimal scheme is Scheme 4. The average annual water shortage of Scheme 4 is small and the average annual water transfer is also at a relatively reasonable level.

Tab. 6 Distance between each scheme and the theoretical optimal/worst solution as well as their relative closeness

Distance and relative closeness	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$d(x_i, x^+)$	0.641 6	1.063 2	1.124 3	0.426 2	1.000 2	1.486 9	1.488 0
$d(x_i, x^-)$	0.783 6	0.644 4	0.605 4	1.133 2	0.930 2	0.832 6	0.858 2
C_i	0.450 2	0.622 6	0.650 0	0.273 3	0.518 1	0.641 0	0.634 2

5 Conclusions

The multi-objective optimal allocation of water resources under the conditions of project water transfer requires overall coordination of the relationship between local water and project water transfer. When selecting the best solution from the Pareto optimal solutions, we evaluate two indices optimized in opposite directions. The distance measure of interval hesitant fuzzy language number is defined and combined with the traditional TOPSIS method. The Pareto optimal set is selected by the multi-index decision-making method based on the interval hesitant fuzzy language set and TOPSIS. The following conclusions can be drawn: (1) After the selection of the allocation schemes of water resources in the northern Hubei, Scheme 4 is chosen as the optimal water allocation scheme. Scheme 4 focuses on reducing water shortage while taking into account water transfer, with high rationality and feasibility, and has high reference value in the actual applications to the allocation projects of water resources in the northern Hubei. (2) The multi-objective decision-making method of water resources based on interval hesitant fuzzy language set and TOPSIS has been proved to have high stability by examples in this paper.

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